TÜRKMENISTANYŇ BILIM MINISTRLIGI

MAGTYMGULY ADYNDAKY TÜRKMEN DÖWLET UNIWERSITETI

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KWANT MEHANIKASY

boýunça amaly usuly gollanma

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Sözbaşy

Türkmenistanyň hormatly Prezidenti Gurbanguly Berdimuhamedowyň gös-göni ýolbaşçylygynda täze Galkynyş we beýik Özgertmeler eýýamynda bilim we ylym ulgamyny döwrebaplaşdyrmak hem-de kämilleşdirmek boýunça örän düýpli çäreler we işler amala aşyrylýar. Diňe 2010-njy ýylyň dowamynda ýokary okuw jaýlarynyň ählisinde olarda geçirilýän dersler boýunça okuw gollanmalary we okuw kitaplary öz ene dilimizde taýýar edildi. Olara mysal edip, Magtymguly adyndaky Türkmen döwlet uniwersitetiniň mugallymlary G.Orazowyň we G.Annamuhammedowyň taýýarlan “kwant mehanikasy” boýunça okuw kitabyny görkezmek bolar. Hödürlenýän meseleler ýygyndysy şol okuw kitabynyň esasynda düzüldi we onuň mazmuny bu ders boýunça geçirilýän umumy okuwlar bilen ýakyn utgaşdyryldy. Talyplaryň özbaşdak işlerini ýeňilleşdirmek maksady bilen her bölümde gerek bolan käbir düşünjeler we formulalar getirildi. Meseleleriň has çylşyrymlysynyň çözgüdi giňişleýin berildi. Türkmen dilinde kwant mehanikasy boýunça şeýle meseleleriň ýygyndysy ilkinjileriň biridir diýsegem bolar. Şu gollanmanyň ilkinji ýazgysyny professor G.Toýlyýew doly okap çykdy, usuly görkezmeleri hödürledi. Onuň bellikleri doly suratda hasaba alyndy. Biz oňa özümiziň minnetdarlygymyzy bildirýäris. Şonuň üçin gollanma barada ýüze çykan teklipleriňizi we bellikleriňizi uniwersitetiñ nazary we eksperimental fizika kafedrasyna ibermegiňizi haýyş edýäris.

I bap

Elektromagnit şöhlelenmäniň kwant nazaryýeti

I.1.Usuly görkezmeler

- M.Plankyň we A.Eýnşteýniň ýagtylyk kwantlary baradaky gipoteza laýyklykda, ýagtylyk diskret bölejikler (kwantlar) arkaly goýberilýär, siňdirilýär we ýaýraýar. Olara fotonlar diýilýär.

- Fotonyň energiýasy:

$$\varepsilon = h\nu = \hbar\omega = \frac{hc}{\lambda} \qquad \textbf{(1)}$$

- Fotonyň massasy energiýanyň we massanyň özara baglanyşyk kanunyndan tapylýar:

$$m = \frac{\varepsilon}{c^2} \qquad \textbf{(2)}$$

- Fotonyň impulsyny otnositelligiň ýörite nazaryýetiniň umumy formulasynda

$$E = c\sqrt{m_0^2 c^2 + P^2},$$

fotonyň dynçlyk massasyny nola deňläp tapylýar:

$$P = \frac{\varepsilon}{c} = \frac{h\nu}{c} = \hbar k, \qquad \textbf{(3)}$$

bu ýerde $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ - tolkun sany.

- (1) we (3) aňlatmalar fotonyň korpuskula häsiýetnamalaryny – masssany, impulsy we energiýany – ýagtylygyň tolkun häsiýeti - ν ýygylyk bilen baglanyşdyrýarlar.

I.2. Meseleler

1.1. Nusgawy elektrodinamikasyna laýyklykda, $\vec{a}$ tizlenme bilen hereket edýän elektron şöhlelenme energiýany

$$\frac{dE}{dt} = -\frac{e^2}{6\pi\varepsilon_0 c^3} a^2 \qquad \textbf{(1)}$$

kanun boýunça ýitirýär. Wodorod atomynyň ýadrosynyň daşynda $r_0 = 0{,}53\,nm$ radiusly töwerek boýunça aýlanýan elektronyň şöhlelenme netijesinde energiýanyň ýitmegi zerarly, ýadro gaçmaklygynyň τ wagtyny kesgitlemeli.

Berlen:

$r_0 = 0{,}53\,nm = 0{,}53 \cdot 10^{-9}\,m$

$m_0 = 9{,}1 \cdot 10^{-31}\,kg$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\,\dfrac{Kl^2}{J \cdot m}$

τ - ?

Çözüwi:

Meseläni sadalaşdyrmak üçin, elektron degişli radiusly töwerek boýunça islendik wagtda deňölçegli hereket edýär diýip hasap ederis. Onda şöhlelenmä garamazdan, elektronyň tizlenmesini

$$a = \frac{\upsilon^2}{r} \qquad \textbf{(2)}$$

formula bilen hasaplap bolýar.

bu ýerde υ - elektronyň hereketiniň tizligi,

r - berlen wagtda elektronyň ýadrodan aralygy.

Merkeze ymtylýan we Kulon güýçleriň deň bolmaklarynyň şerti

$$m_0 \frac{\upsilon^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad \textbf{(3)}$$

Şu ýerden kinetik energiýa

$$E_k = \frac{m_0\upsilon^2}{2} = \frac{e^2}{8\pi\varepsilon_0 r}$$

alynýar we ýadronyň meýdanynda elektronyň doly energiýasy

$$E = E_k + E_p = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r} \qquad \textbf{(4)}$$

deňdir.

Şöhlelenme ýitýän energiýany tapmak üçin, (4)-i wagt boýunça differensirläliň:

$$\frac{dE}{dt} = -\frac{e^2}{8\pi\varepsilon_0} \cdot \frac{d}{dt}\left(\frac{1}{r}\right) = \frac{e^2}{8\pi\varepsilon_0} \cdot \frac{1}{r^2}\frac{dr}{dt} \qquad \textbf{(5)}$$

(1) we (5)-i deňeşdirip we (2) we (3)-i hasaba alyp, alarys:

$$\frac{e^2}{8\pi\varepsilon_0}\frac{1}{r^2}\frac{dr}{dt} = -\frac{1}{96}\left(\frac{e^2}{\pi\varepsilon_0 c}\right)^3 \frac{1}{m_0^2 r^4},$$

şu ýerden

$$-r^2 dr = \frac{1}{12c^3}\left(\frac{e^2}{\pi\varepsilon_0 m_0}\right)^2 dt.$$

Şu deňlemäni r boýunça $r_0 - dan$ $0 - a$ çenli we t boýunça $0 - dan$ τ (elektronyň orbitasynyň $r_0 - dan$ $0 - a$ çenli kiçelmeginiň wagty) çenli integrirläliň:

$$-\int_{r_0}^{0} r^2 dr = \frac{1}{12c^3}\left(\frac{e^2}{\pi\varepsilon_0 m_0}\right)^2 \int_0^{\tau} dt.$$

Şu ýerden

$$\tau = 4c^3\left(\frac{\pi\varepsilon_0 m_0}{e^2}\right)^2 r_0^3 = 14 \cdot 10^{-12}\, s = 14\, ps$$

1.2. Fotony şöhlelendirip, dynçlykdaky wodorodyň atomy birinji oýandyrylan ýagdaýdan esasy ýagdaýa geçýär. Atomyň gaýtargy tizligini kesgitlemeli.

Berlen:

$\hbar = 1{,}05 \cdot 10^{-34}\, J \cdot s$

$R = 1{,}097 \cdot 10^7\, m^{-1}$

$m = 1{,}0078\, m.a.b = 1{,}68 \cdot 10^{-27}\, kg$

ϑ -?

Çözüwi:

Impulsyň saklanma kanunyna laýyklykda, atom ondan uçan fotonyň impulsyna deň impulsa eýe bolýar, ýagny

$$m\upsilon = \frac{\hbar\omega}{c} \qquad \textbf{(1)}$$

bu ýerde m we ϑ - atomyň massasy we tizligi,

ω - fotonyň ýygylygy

Energiýanyň saklanma kanunyna laýyklykda, atomyň oýandyrylan energiýasy,

$$\Delta E = \hbar R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}\hbar R \qquad \textbf{(2)}$$

fotonyň energiýasynyň we gaýtarga sezewar bolan atomyň kinetik energiýasynyň arasynda paýlanýar, ýagny

$$\Delta E = \hbar\omega + \frac{m\upsilon^2}{2} \qquad \textbf{(3)}$$

deňlik ýerine ýetýär.

(1) we (2)-ni (3)-e goýup

$$\frac{3}{4}\hbar R = mc\upsilon + \frac{m\upsilon^2}{2}$$

alynýar. Şu taýdan atomyň gaýtargy tizligi alynýar:

$$\upsilon = \sqrt{c^2 + \frac{3}{2}\frac{\hbar R}{m}} - c = 3{,}27\ \frac{m}{s}.$$

1.3. Dynçlykdaky proton özünden daşda $1{,}875 \cdot 10^8 \frac{m}{s}$ tizlik bilen hereket edýän elektrony gabap alýar we netijede wodorodyň oýandyrylan atomy emele gelýär. Esasy ýagdaýa geçende atomyň goýberýän fotonynyň tolkun uzynlygyny kesgitlemeli.

Berlen:	**Çözüwi:**
$\upsilon = 1{,}875 \cdot 10^6 \frac{m}{s}$ $\nu = 6{,}63 \cdot 10^{-34}\ J \cdot s$ $m_0 = 9{,}1 \cdot 10^{-31}\ kg$ ——— λ - ?	Goýberilen fotonyň tolkun uzynlygy deňdir: $$\lambda = \frac{c}{\nu} = \frac{hc}{h\nu} = \frac{hc}{\Delta E} \qquad \textbf{(1)}$$ bu ýerde ΔE - esasy ýagdaýa atom geçende bölünýän energiýa.

Energiýanyň saklanma kanunyna laýyklykda, ΔE protondan daşda ýerleşen elektronyň kinetik energiýasyna deňdir, ýagny

$$E = \frac{m_0 \upsilon^2}{2} \qquad \textbf{(2)}$$

(2)-ni (1)-e goýup

$$\lambda = \frac{2hc}{m_0 \upsilon^2} = 5{,}25 \cdot 10^{-8}\, m.$$

1.4. $1{,}02\, MeW$ kinetik energiýaly elektronyň massasy onuň dynçlyk massasyndan näçe esse uly?

Çözülişi:

Elektronyň doly energiýasy onuň massasyna göni proporsionaldyr:

$$E = mc^2, \qquad \textbf{(1)}$$

bu ýerde m - hereket edýän elektronyň massasy,
c - wakuumda ýagtylygyň tizligi.

Elektronyň kinetik energiýasy E_k onuň doly energiýasynyň E we dynçlyk energiýasynyň E_o tapawudy ýaly kesgitlenilýär:

$$E_k = E - E_o \qquad \textbf{(2)}$$

bu ýerde

$E_o = m_o c^2$, m_o - elektronyň dynçlyk massasy.

Onda

$$E_k = m_o c^2 \left(\frac{m}{m_o} - 1 \right),$$

şu ýerden

$$\frac{m}{m_o} = \frac{E_k}{m_o c^2} + 1 \qquad \textbf{(3)}$$

Ululyklaryň bahalaryny goýup, alarys:

$$\frac{m}{m_0} = \frac{1{,}02\, MeW}{0{,}51\, MeW} + 1 = 3.$$

Şeýlelikde, $1{,}02\, MeW$ kinetik energiýaly elektonyň massasy onuň dynçlyk massasyndan 3 esse uly.

II bap

M.Plankyň formulasy. Fotoeffekt. Komptonyň effekti

II.1. Usuly görkezmeler

- Absolýut gara jisimiň deňagramly şöhlelenmesiniň spektral dykyzlygy üçin M.Plank şeýle formulany alypdyr:

$$\rho_\omega = \frac{\hbar}{\pi^2 c^3} \cdot \frac{\omega^3}{e^{\frac{\hbar\omega}{kT}} - 1}$$

- Kwant düşünjesiniň kömegi bilen A.Eýnşteýn daşky fotoeffekt hadysasyny düşündiripdir we şeýle deňlemäni alypdyr:

$$\frac{m_0 \upsilon_m^2}{2} = \hbar\omega - A$$

Görnüşi ýaly, fotoelektronyň maksimal kinetik energiýasy $\frac{m_0 \upsilon_m^2}{2}$, fotonyň $\hbar\omega$ energiýasy we elektronyň metaldan A çykyş işiniň tapawudyna deňdir.
Fotonyň impulsynyň barlygyny Komptonyň effekti subut edýär. Rentgen şöhlesiniň erkin elektronlarda pytranda onuň tolkun uzynlygynyň üýtgemegine ***Kompton effekti*** diýilýär.

- Tolkun uzynlygynyň $\Delta\lambda$ üýtgemesi aşakdaky formula bilen aňladylýar:

$$\Delta\lambda = \lambda' - \lambda = 2\lambda_0 \sin^2\frac{\theta}{2}.$$

bu ýerde,

$\lambda_0 = \frac{2\pi\hbar}{m_0 c} = 2,4263 \cdot 10^{-12}\, m$ - Kompton tolkun uzynlygy;

θ - düşýän şöhläniň pytrama burçy.

II.2.Meseleler

2.1. $\hbar\omega >> kT$ we $\hbar\omega << kT$ iki çäkli ýagdaýlarda deňagramly şöhlelenmäniň energiýasynyň spektral dykyzlygy üçin ýakynlaşan aňlatmalary almaly.

Çözülişi:

Deňagramly şöhlelenmäniň spektral dykyzlygynyň formulasy

$$\rho_\omega = \frac{\hbar\omega^3}{\pi^2 c^3\left(e^{\frac{\hbar\omega}{kT}} - 1\right)}$$ - ***M.Plankyň formulasy***. **(1)**

Uly bolmadyk ýygylyklar $\frac{\hbar\omega}{kT} << 1$ üçin $e^{\frac{\hbar\omega}{kT}} \approx 1 + \frac{\hbar\omega}{kT}$.

Onda (1) aşakdaky görnüşe geçýär.

$$\rho_\omega = \frac{\omega^2}{\pi^2 c^3} kT$$ - ***Releýiň- Jinsiň formulasy***.

Uly ýygylyklar $\left(\frac{\hbar\omega}{kT} >> 1\right)$ üçin şeýle şert emele gelýär $e^{\frac{\hbar\omega}{kT}} >> 1$.

Şeýle ýagdaýda (1)-de ýaýyň içindäki 1-ligi inkär edip, alýarys:

$$\rho_\omega = \frac{\hbar\omega^3}{\pi^2 c^3} \cdot e^{-\frac{\hbar\omega}{kT}}$$ - ***eksponensial kanuny***.

2.2. Plankyň formulasynyň kömegi bilen, absolýut gara jisim üçin Stefanyň- Bolsmanyň kanunyny getirip çykarmaly we ondaky hemişeligiň san bahasyny hasaplamaly.

Çözülişi:

Plankyň formulasy $$\rho_\omega = \frac{\hbar}{\pi^2 c^3} \cdot \frac{\omega^3}{e^{\frac{\hbar\omega}{kT}} - 1}$$ **(1)**

Elektromagnit energiýanyň adaty dykyzlygy:

$$u = \int_0^\infty \rho_\omega d\omega$$ **(2)**

(1)-i (2)-de ornuna goýalyň:

$$u = \frac{\hbar}{\pi^2 c^3} \int_0^\infty \frac{\omega^3}{e^{\frac{\hbar\omega}{kT}} - 1} d\omega$$

Täze üýtgeýän ululyk girizýäris:

$$\xi = \frac{\hbar\omega}{kT}.$$

Şu ýerden

$$d\omega = \frac{kT}{\hbar} d\xi$$

Onda

$$u = \frac{k^4 T^4}{\hbar^3 \pi^2 c^3} \cdot \int_0^\infty \frac{\xi^3}{e^\xi - 1} d\xi$$

Bu ýerde,

$$\int_0^\infty \frac{\xi^3}{e^\xi - 1} d\xi = \frac{\pi^4}{15},$$

Şeýlelikde

$$u = aT^4,$$

bu ýerde

$$a = \frac{\pi^2 k^4}{15 c^3 \hbar^3} = 7.56 \cdot 10^{-28} J \cdot m^{-3} grad^{-4}.$$

2.3. Plankyň formulasyny ulanyp Winiň süýşme kanunyny getirip çykarmaly we onuň hemişeligini hasaplamaly.

Çözüwi:
Plankyň formulasyny tolkun uzynlygynyň üsti bilen aňladalyň.

$$\omega = \frac{2\pi c}{\lambda}$$

we

$$d\omega = -2\pi c \frac{d\lambda}{\lambda^2}.$$

Ýene-de

$$u = \int_0^\infty \rho_\lambda d\lambda \quad ,$$

aňlatmany ulanyp, taparys:

$$\int_0^\infty \rho_\lambda d\lambda = u = \int_0^\infty \rho_\omega d\omega = -2\pi c \int_0^\infty \rho_\omega \frac{d\lambda}{\lambda^2} = 2\pi c \int_0^\infty \rho_\omega \frac{d\lambda}{\lambda^2}$$

ýa-da

$$\int_0^\infty \left\{ \rho_\lambda - 2\pi c \rho_\omega \frac{d\lambda}{\lambda^2} \right\} d\lambda = 0$$

ýa-da

$$\rho_\lambda = \frac{2\pi c}{\lambda^2} \rho_\omega ,$$

ýa-da

$$\rho_\lambda = \frac{16 \hbar \pi^2 c}{\lambda^5 \left(e^{\frac{2\pi c\hbar}{kT\lambda}} - 1 \right)} \quad .$$

ρ_λ dykyzlygyň maksimal bahasyna degişli λ_{max} bahany kesgitlemek üçin, ýokarky aňlatmadan $\frac{\partial \rho_\lambda}{\partial \lambda} = 0$ ululygy tapmaly:

$$\left[-5 + \frac{\frac{2\pi c\hbar}{kT\lambda_{max}} \cdot e^{\frac{2\pi c\hbar}{kT\lambda_{max}}}}{e^{\frac{2\pi c\hbar}{kT\lambda_{max}}} - 1} \right] = 0 \, .$$

$\frac{2\pi c\hbar}{kT\lambda_{max}} = y$ belgilemäni girizip, alarys:

$$y = 5\left(1 - e^{-y}\right).$$

Şu deňlemäniň çözgüdi uly takyklyk bilen şeýle görnüşde alnyp bilner.

$$y \approx 5\left(1 - e^{-5}\right) \approx 4.965$$

Onda

$\lambda_{max} T = b$ - *Winiň süýşme kanuny.*

Bu ýerde,

$$b = \frac{2\pi c\hbar}{4.965 \cdot k} = 0.29 \cdot 10^{-2}\, m \cdot grad$$ ***- Winiň hemişeligi***

2.4. Seziý we platina üçin fotoeffektiň gyzyl araçäginiň tolkun uzynlygyny hasaplamaly. Olardan elektronlary goparmak üçin iş degişlilikde 1,89 eW we 5,29 eW .

Jogaby: $\lambda_{sz} = 657\ nm, \quad \lambda_{pt} = 235\ nm.$

2.5. Erkin elektronda fotoeffektiň bolmajagyny görkezmeli.

Çözüwi:

Meseläni tersine güman etmek bilen çözeliň. Goý, erkin elektron fotonyň energiýasyny doly siňdirýär diýeliň. Şeýle ýagdaý üçin energiýanyň we impulsyň saklanma kanunlaryny ýazalyň.

a). Relýatiwistik däl ýagdaýda

$$\varepsilon_\gamma = \frac{m\upsilon^2}{2},$$

we

$$\frac{\varepsilon_\gamma}{c} = m\upsilon .$$

Şu iki deňlemelerden $\upsilon = 2c$ gelip çykýar, emma bu mümkin däldir.

b). Relýatiwistik ýagdaýda

$$\varepsilon_\gamma = m_o c^2 \left[\frac{1}{\sqrt{1-\beta^2}} - 1 \right],$$

we

$$\frac{\varepsilon_\gamma}{c} = \frac{m_o c\beta}{\sqrt{1-\beta^2}} .$$

Soňky iki deňlemelerden

$$\left(1-\beta^2\right) = 1-\beta^2$$

ýa-da

$$\beta(\beta-1)=0\,.$$

Bu ýerden $\beta=0$ we $\beta=1$ - bu hem bolup bilmez.

2.6. $0{,}35\,MeW$ energiýaly foton erkin elektronda 60^0 burç bilen dargaýar. Elektronyň başlangyç energiýasyny hasaba alman, dargan fotonyň ε' energiýasyny we “depilen”elektronyň impulsyny tapmaly.

Berlen:

$\varepsilon=0.35\,MeW$

$\alpha=60^0$

$\varepsilon'-?$

$P-?$

Çözüwi:

Energiýanyň saklanma kanuny boýunça

$$\varepsilon=\varepsilon'+E_{kin}\ ,$$

bu ýerde

$$\varepsilon'=\hbar\omega'=\frac{2\pi c\hbar}{\lambda'}$$

Komptonyň formulasyndan

$$\lambda'=\lambda+2\lambda_0\sin^2\frac{\theta}{2}\,,$$

$$\lambda_0=\frac{2\pi\hbar}{m_0c}\,.$$

Onda

$$\varepsilon'=\frac{2\pi c\hbar}{\lambda+2\lambda_0\sin^2\frac{\theta}{2}}=\frac{\frac{2\pi c\hbar}{\lambda}}{1+\frac{2}{m_0c^2}\cdot\frac{2\pi c\hbar}{\lambda}\sin^2\frac{\theta}{2}}=\frac{\varepsilon}{1+2\frac{\varepsilon}{m_0c^2}\sin^2\frac{\theta}{2}}$$

nirede

$$m_0c^2=0.51\,MeW$$

Şeýlelikde,

$$\varepsilon' = \frac{0{,}35\,MeW}{1+2\dfrac{0{,}35\,MeW}{0{,}51\,MeW}\cdot\dfrac{1}{4}} = 0{,}26\,MeW \quad .$$

Elektronyň kinetik energiýasy $E_{kin} = \varepsilon - \varepsilon' = 0{,}09\,MeW$.

Impulsy kinetik energiýanyň üsti bilen aňladyp alarys.

$$P = \frac{1}{c}\sqrt{E_{kin}\left(E_{kin} + 2m_0c^2\right)} \approx 1{,}68\cdot 10^{-27}\,\frac{kg\cdot m}{s} \,.$$

Jogaby: $\varepsilon' = 0{,}25\,MeW$, $P = 1{,}65\cdot 10^{-27}\,\dfrac{kg\cdot m}{s}$.

2.7. Relýatiwistik elektron bilen çaknyşanda θ burça pytraýar, elektron bolsa durýar. Pytran fotonyň tolkun uzynlygynyň Kompton süýşmesini tapmaly.

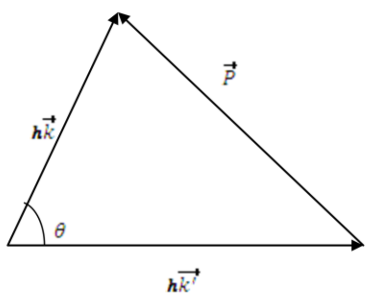
Çözüwi:

Impulsyň saklanma kanunyna görä,

$$\hbar\vec{k} + \vec{P} = \hbar\vec{k}',$$

bu ýerde, $\vec{k}$ we $\vec{k}'$ - başdaky we pytran fotonlaryň tolkun wektorlary,

$\vec{P}$ - elektronyň impulsy.



Çyzgydan kosinuslar teoremasynyň esasynda alynýar:

$$\vec{P}^2 = \left(\hbar\vec{k}\right)^2 + \left(\hbar\vec{k}'\right)^2 - 2\hbar^2\vec{k}\vec{k}'\cos\theta \qquad \textbf{(1)}$$

Eger (1)-de

$$\vec{k} = \frac{\omega}{c}, \qquad \vec{k}' = \frac{\omega'}{c}, \qquad \varepsilon = \hbar\omega \quad \text{we} \quad \varepsilon' = \hbar\omega'$$

aňlatmalar hasaba alynsa, onda alarys:

$$\vec{P}^2 = \frac{1}{c^2}\left(\varepsilon^2 + \varepsilon_1^2 - 2\varepsilon\varepsilon'\cos\theta\right) \qquad \textbf{(2)}$$

Energiýanyň saklanma kanuny:

$$\varepsilon + E = \varepsilon' + m_0c^2,$$

bu ýerde E - elektronyň doly energiýasy,

m_0 - onuň dynçlyk massasy.

Soňky aňlatmadan

$$E^2 = \varepsilon^2 + \varepsilon'^2 + m_0^2c^4 - 2\varepsilon\varepsilon' + 2m_0c^2\left(\varepsilon' - \varepsilon\right) \qquad \textbf{(3)}$$

Indi $m_0c^2 = \sqrt{E^2 - P^2c^2}$ gatnaşygyň iki tarapyny kwadrata götereliň we (2) we (3)-i hasaba alalyň:

$$\varepsilon\varepsilon'(1 - \cos\theta) = m_0c^2(\varepsilon' - \varepsilon),$$

ýa-da

$$\frac{\hbar}{m_0c^2}(1 - \cos\theta) = \frac{1}{\omega} - \frac{1}{\omega'} = \frac{\lambda - \lambda'}{2\pi c},$$

Şu ýerden

$$\lambda - \lambda' = -\frac{4\pi\hbar}{m_0c}\sin^2\frac{\theta}{2} < 1$$

alynýar.

Görnüşi ýaly, pytran tolkun uzynlygy kiçi bolýar we onuň energiýasy artýar.

2.8. Protonyň kinetik energiýasy özüniň dynçlyk energiýasyndan 4 esse kiçi. Protonyň de Broýl tolkun uzynlygyny hasaplamaly.

Berlen:	**Çözüwi:**
$E_k = \frac{E_0}{4}$ $E_0 = 1{,}5 \cdot 10^{-10}\ J$ λ - ?	Lui de Broýlyň tolkun uzynlygy

$$\lambda = \frac{2\pi\hbar}{P} \qquad \textbf{(1)}$$

formula boýunça hasaplanýar. Impuls we kinetik energiýa

$$P = \frac{1}{c}\sqrt{E_k\left(E_k + 2m_0c^2\right)}, \qquad \textbf{(2)}$$

gatnaşyk (2.6 - meselä seret) bilen baglanyşykly.

Meseläniň şerti

$$E_k = \frac{E_0}{4} \qquad \textbf{(3)}$$

(3)-i (2)-ä goýalyň:

$$P = \frac{1}{c}\sqrt{\frac{E_0}{4}\left(\frac{E_0}{4} + 2E_0\right)} = \frac{3}{4}\cdot\frac{E_0}{c} \qquad \textbf{(4)}$$

(4)-i (1)-e goýalyň:

$$\lambda = \frac{8\pi\hbar c}{3E_0} = \frac{4hc}{3E_0} = \frac{4\cdot 6{,}63\cdot 10^{-34\cdot 3\cdot 10^{8}}}{\sqrt{3}\cdot 1{,}5\cdot 10^{-10}}\,m = 1{,}77\cdot 10^{-15}\,m.$$

2.9. $\varepsilon = 0{,}75\,MeW$ energiýaly foton erkin elektronda 60^0 burç bilen dargaýar. Elektronyň urgudan öňki energýasyny hasaba alman, dargan fotonyň energiýasyny we elektronyň kinetik energiýasyny tapmaly.

Jogaby: $\varepsilon' \approx 0{,}43\,MeW$, $E_{kin} = 0{,}32\,MeW$.

2.10. Eger kümüşden elektrony goparmak üçin zerur iş *A=4,28eW* bolsa,onda fotoeffektiň gyzyl araçägine degişli tolkun uzynlygyny hasaplamaly.

Jogaby: $\lambda = \frac{2\pi\hbar}{A} = 290\ nm.$

2.11. Düşýän şöhleler dessesiniň ugruna perpendikulýar seredilen ýagdaýynda Komptonyň effektindäki tolkun uzynlygynyň üýtgemegini kesgtilemeli.

Jogaby: $\Delta\lambda = \frac{4\pi\hbar}{m_0 c}\sin^2\frac{\theta}{2} = 2{,}4263 \cdot 10^{-3}\ nm.$

2.12. $2{,}2 \cdot 10^{15}\ s^{-1}$ ýygylyga eýe bolan monohromatik ýagtylyk bilen metalyň üstünden goparylýan fotoelektronlar $-6{,}6W$ potensial bilen doly suratda saklanylýarlar, $4{,}6 \cdot 10^{15}\ s^{-1}$ ýygylykly monohromatik ýagtylygyň goparýan elektronlary bolsa *-16,5W* potensial bilen saklanylýarlar. Bu maglumatlar boýunça Plankyň hemişeligini kesgitlemeli.

Berlen:

$\nu_1 = 2{,}2 \cdot 10^{15}\ s^{-1}$

$\nu_1 = 4{,}6 \cdot 10^{15}\ s^{-1}$

$\varphi_1 = -6{,}6W$

$\varphi_2 = -16{,}5W$

$\hbar - ?$

Çözüwi:

Plankyň hemişeligini fotoeffekt üçin Eýnşteýniň deňlemesinden kesgitläp bolýar:

$$\varepsilon = A + E_k, \qquad \textbf{(1)}$$

bu ýerde ε – fotonyň energiýasy,

A – çykyş işi,

E_k – fotoelektronlaryň maksimal kinetik energiýasy

Fotonyň energiýasy şeýle formula boýunça hasaplanylýar:

$$\varepsilon = h\nu$$

bu ýerde h - Plankyň hemişeligi,

ν - düşýän ýagtylygyň ýygylygy

Elektronyň kinetik energiýasy saklaýjy potensialyň işine deňdir:

$$E_k = e\varphi\ ,$$

bu ýerde e - elektronyň zarýady,

φ - saklaýjy potensial.

Onda (1)-njiniň esasynda ýazyp bileris:

$$h\nu_1 = A + e\varphi_1$$
$$h\nu_2 = A + e\varphi_2$$

Şu iki deňlemelerden

$$h = \frac{e(\varphi_2 - \varphi_1)}{\nu_2 - \nu_1}.$$

Ululyklaryň bahalaryny orunlaryna goýup alarys:

$$h = \frac{1.6\cdot 10^{-19}\,Kl\,(16.5W - 6.6W)}{4.6\cdot 10^{15}\,s^{-1} - 2.2\cdot 10^{15}\cdot s^{-1}} = 6.6\cdot 10^{-34}\,J\cdot s$$

$$\hbar = \frac{h}{2\pi} = \frac{6{,}6\cdot 10^{-34}}{6{,}28}\,J\cdot s = 1{,}05\cdot 10^{-34}\,J\cdot s.$$

Jogaby: $\hbar = 1{,}05\cdot 10^{-34}\,J\cdot s$

2.13. 21.4 pm tolkun uzynlykly rentgen şöhlesiniň fotony Kompton effekti netijesinde başlangyç ugra 90^0 burç bilen dargaýar. Foton özüniň energiýasynyň näçe bölegini elektrona berýär?

Berlen:	**Çözüwi:**
$\lambda_1 = 21.4\,pm = 21.4\cdot 10^{-12}\,m$ $\theta = 90^0$ $n - ?$	Komptonyň formulasy

$$\Delta\lambda = \lambda' - \lambda = 2\lambda_0 \sin^2\frac{\theta}{2},$$

bu ýerde

$$\lambda_0 = \frac{2\pi\hbar}{m_0 c} = 2.43\cdot 10^{-12}\,m$$ - ***Kompton tolkun uzynlygy***

Fotonyň energiýasy

$$\varepsilon = \frac{2\pi c\hbar}{\lambda}$$

Dargan fotonyň energiýasy:

$$\varepsilon = \frac{2\pi c\hbar}{\lambda'}$$

Elektrona berlen energiýa düşýän fotonyň energiýasynyň kesgitli bölegini düzýär:

$$n = \frac{\varepsilon - \varepsilon'}{\varepsilon} = \frac{\lambda' - \lambda}{\lambda'}$$

ýa-da

$$n = \frac{2\lambda_0 \sin^2\frac{\theta}{2}}{\lambda + 2\lambda_0 \sin^2\frac{\theta}{2}}$$

Ululyklaryň bahalaryny ounlaryna goýup, alarys:

$$n = \frac{2 \cdot 2.43 \cdot 10^{-12}\, m \cdot \frac{1}{2}}{21.4 \cdot 10^{-12}\, m + 2 \cdot 2.43 \cdot 10^{-12}\, m \cdot \frac{1}{2}} = 0.102 \ .$$

Jogaby: Rentgen fotony elektronda darganda elektrona özüniň energiýasynyň 10,2% berýär.

III bap

Mikrobölejikleriň tolkun häsiýeti. Kesgitsizlik gatnaşygy

III.1. Usuly görkezmeler

- N.Bor üç fiziki düşünjeleri (atom, elektron, şöhlelenme) díňe bír kwant düşünjesiniň üsti bilen özara baglanyşdyrypdyr. Onuň birinji postulaty

$$m\vartheta r = n\hbar\,, \qquad n = 1,2,3,...$$

- Boruň nazaryýetine laýyklykda, stasionar ýagdaýlary, adiabatik inwariantlary kwantlandyrmak ýoly bilen kesgitläp bolýar:

$$\oint P_i dq_i = n\hbar$$

- Lui de Broýlyň gipotezasy boýunça erkin hereket edýän bölejige tekiz monohromatik tolkun degişlidir we onuň tolkun uzynlygy

$$\lambda = \frac{2\pi\hbar}{P}$$ - ***de Broýlyň formulasy***

- Mikrobölejikleriň korpuskulýar - tolkun tebigaty, ýagny dualizm häsiýeti, Geýzenbergiň kesgitsizlik gatnaşygy bilen suratlandyrylýar.

  Bölejikleriň koordinatalary we impulsy üçin

$$\Delta x \cdot \Delta P_x \geq \hbar, \qquad \Delta y \cdot \Delta P_y \geq \hbar, \qquad \Delta z \cdot \Delta P_z \geq \hbar.$$

bu ýerde, $\Delta x,\ \Delta y,\ \Delta z$ - bölejigiň koordinatasynyň kesgitsizligi;

$\Delta P_x,\ \Delta P_y,\ \Delta P_z$ - degişli impulsyň düzüjisiniň kesgitsizligi;

Energiýa we wagt üçin

$$\Delta E \cdot \Delta t \geq \hbar,$$

bu ýerde ΔE - berlen kwant ýagdaý üçin energiýanyň kesgitsizligi;

Δt - şol ýagdaýda sistemanyň bolmaklygynyň wagty.

III.2. Meseleler

3.1. Boruň nazaryýetini ulanyp, wodorod atomynyň orbitasynyň radiusyny, ondaky elektronyň tizligini kesgitlemeli.

Berlen:	**Çözüwi:**
$m = 9{,}1 \cdot 10^{-31}\, kg$ $e = 1{,}6 \cdot 10^{-19}\, Kl$ $\hbar = 1{,}05 \cdot 10^{-34}\, J \cdot s$	
$\upsilon_1 - ?$ $r_1 - ?$	

Boruň birinji postulatyna laýyklykda

$$m\upsilon_n r = n\hbar\,, \qquad n = 1,2,3,... \tag{1}$$

Kulon güýji merkeze ymtylýan güýç bolup hyzmat edýär.

$$k\frac{ze \cdot e}{r_n^2} = \frac{m\upsilon_n^2}{r_n} \quad \text{ýa – da} \quad m\upsilon_n^2 r_n = kze^2 \tag{2}$$

(1) we (2) deňlikleri bölüp alarys:

$$\upsilon_n = \frac{kze^2}{n\hbar} \tag{3}$$

(3) - i (1) – e goýup, alarys:

$$r_n = \frac{n^2 h^2}{kmze^2}$$

Meseläniň şertine görä n=1, z=1, şonuň üçin

$$\upsilon_1 = \frac{ke^2}{\hbar} = 2{,}183 \cdot 10^6\,\frac{m}{s} \quad \text{we} \quad r_1 = \frac{\hbar}{kme^2} = 0.53 \cdot 10^{-10}\, m.$$

Jogaby: $\upsilon_1 = 2{,}183 \cdot 10^6\,\frac{m}{s}$,

$r_1 = 0{,}53 \cdot 10^{-10}\, m.$

3.2. Erkin bölejik tükeniksiz beýik diwarly potensial çukuryň $x = 0$ we $x = a$ nokatlarynda ýerleşýär. Kwantlanma düzgünini ulanyp, bölejigiň stasionar ýagdaýynyň energiýasyny kesgitlemeli.

Çözüwi.

Çukuryň içinde bölejik hereket edende onuň “P” impulsy hemişelik bolup galýar, ýöne diwardan serpigende diňe ugry boýunça üýtgeýär. Impulsyň ululygyny kwantlanma düzgüninden tapýarys.

$$\oint P_x dx = n\hbar\,, \qquad n = 1,2,3,... \tag{1}$$

Seredilýän ýagdaýda

$$\oint P_x dx = \int_0^a Pdx + \int_a^0 (-P)dx = \int_0^a Pdx + \int_0^a Pdx = 2Pa \tag{2}$$

Diýmek, bolup biljek impulsyň bahalaryny (1) we (2)-den alarys;

$$P_n = \frac{\hbar}{2a}n$$

“m” massaly relýatiwistik däl bölejigiň energiýasy

$$E_n = \frac{P_n^2}{2m} = \frac{\hbar^2}{8a^2m}n^2\,.$$

Görnüşi ýaly, energiýa diskret bahalary alýar.

3.3. Relýatiwistik bölejigiň kinetik energiýasy E_{kin}, onuň dynçlyk massasy bolsa m_0. Şu bölejigiň de Broýl tolkun uzynlygyny tapmaly?

Çözüwi:

De Broýluň tolkun uzynlygy aşakdaky deňlemeden kesgitlenýär:

$$\lambda = \frac{2\pi\hbar}{P} \tag{1}$$

Relýatiwistik bölejik üçin

$$P = \frac{m_0 c\beta}{\sqrt{1-\beta^2}}\,,$$

onda

$$\lambda = \frac{2\pi\hbar\sqrt{1-\beta^2}}{m_0 c\beta}\,.$$

"β" ululygy kinetik energiýanyň aňlatmasyndan tapýarys:

$$E_{kin} = m_0c^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right],$$

ýa – da

$$E_{kin} + m_0c^2 = \frac{m_0c^2}{\sqrt{1-\beta^2}}$$

Şu ýerden

$$\sqrt{1-\beta^2} = \frac{m_0c^2}{E_{kin} + m_0c^2} \qquad \textbf{(2)}$$

Iki tarapyny-da kwadrata götereliň:

$$1-\beta^2 = \frac{m_0^2c^4}{\left(E_{kin} + m_0c^2\right)^2},$$

Ýönekeý özgertmeleri ulanyp alarys:

$$\left(E_{kin} + m_0c^2\right)^2 - m_0^2c^4 = \beta^2\left(E_{kin} + m_0c^2\right)^2,$$

$$E_{kin}^2 + 2E_{kin}m_0c^2 + m_0^2c^4 - m_0^2c^4 = \beta^2\left(E_{kin} + m_0c^2\right)^2,$$

$$\beta = \frac{\sqrt{E_{kin}(E_{kin} + 2m_0c^2)}}{E_{kin} + m_0c^2} \qquad \textbf{(3)}$$

(2) we (3) aňlatmalary (1)-e goýup, taparys:

$$\lambda = \frac{2\pi\hbar c}{\sqrt{E_{kin}\left(E_{kin} + 2m_0c^2\right)}}\ , \qquad \textbf{(4)}$$

Iki ýagdaýa seredeliň:

1. Eger, $E_{kin} << m_0c^2$, ýagny $\vartheta << c$ bolsa (relýatiwistik däl bölejik), onda

$$\lambda = \frac{2\pi\hbar}{\sqrt{2m_0E_{kin}}},$$

bu ýerde,

$$E_{kin} = \frac{m_0\upsilon^2}{2}$$

Dogrudanam, $E_{kin} = m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}} - 1 \right)$ gatnaşykda

$\frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}} = \left(1-\frac{\upsilon^2}{c^2}\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}\frac{\upsilon^2}{c^2} + \ldots$ aňlatmany hasaba alsak, onda

$$E_{kin} = m_0 c^2 \left(1 + \frac{1}{2}\frac{\upsilon^2}{c^2} - 1\right) = \frac{m_0 \upsilon^2}{2}.$$

Eger $E_{kin} >> m_0 c^2$ bolsa (ultrarelýatiwistik bölejik),

onda

$$\lambda = \frac{2\pi\hbar c}{E_{kin}}$$

Meselem, $E_{kin} = 10\,GeW$ bolanda $\lambda = 1{,}6 \cdot 10^{-16}\, m$.

3.4. *λ = 0,2nm de Broýl* tolkun uzynlykly neýtronyň energiýasyny tapmaly. Ýylylyk deňagramlygynda nähili temperaturada neýtronyň energiýasy şeýle baha eye bolup biler?

Berlen:

$\lambda = 0{,}2nm = 0{,}2 \cdot 10^{-9}\, m$

$m = 1{,}675 \cdot 10^{-27}\, kg$

$k = 1{,}38 \cdot 10^{-23} \frac{J}{K}$

$\hbar = 1{,}05 \cdot 10^{-34}\, J \cdot s$

$E - ?$

$T - ?$

Çözüwi:

Belli bolşy ýaly, $E = \frac{P^2}{2m}$ we $P = \frac{2\pi\hbar}{\lambda}$

Onda $E = \frac{2\pi^2\hbar^2}{m\lambda} = 328{,}78 \cdot 10^{-23}\, J = 2{,}5 \cdot 10^{-2}\, eW$

Şerte görä $E = \frac{3}{2}kT$

Bu ýerden

$$T = \frac{4\pi^2\hbar^2}{3km\lambda^2} = 1{,}6 \cdot 10^2\, K$$

Jogaby: $E = 2{,}5 \cdot 10^{-2}\, eW$, $T = 1{,}6 \cdot 10^2\, K$

3.5. Relýatiwistik bölejigiň *de Broýl* tolkun uzynlygy λ we onuň dynçlyk massasy m_0. Onuň kinetik energiýasyny tapmaly.

Berlen:

λ

m_0

$E_{kin} - ?$

Çözüwi:

Kinetik energiýa $$E_{kin} = m_0 c^2 \left[\frac{1}{\sqrt{1-\beta^2}} - 1 \right], \qquad \textbf{(1)}$$

impuls

$$P = \frac{m_0 c \beta}{\sqrt{1-\beta^2}} \qquad \textbf{(2)}$$

formulalardan tapylýar.

Başga tarapdan $$P = \frac{2\pi\hbar}{\lambda} \qquad \textbf{(3)}$$

(2) we (3) aňlatmalaryň sag taraplaryny deňläp, taparys:

$$\beta^2 = \frac{4\pi^2\hbar^2}{4\pi^2\hbar^2 + m_0^2 c^2 \lambda^2} \qquad \textbf{(4)}$$

(4) – i (l) –de goýup, alýarys:

$$E = \frac{m_0 c^2}{\sqrt{1 - \dfrac{4\pi^2\hbar^2}{4\pi^2\hbar^2 + m_0^2 c^2 \lambda^2}}} - m_0 c^2 = \frac{m_0 c^2}{\sqrt{\dfrac{m_0^2 c^2 \lambda^2}{4\pi^2\hbar^2 + m_0^2 c^2 \lambda^2}}} - m_0 c^2 = \sqrt{\frac{m_0^2 c^4 \left(4\pi^2\hbar^2 + m_0^2 c^2 \lambda^2\right)}{m_0^2 c^2 \lambda^2}} - m_0^2 c^2 =$$

$$= c\sqrt{m_0^2 c^2 + 4\pi^2 \left(\frac{\hbar}{\lambda}\right)^2} - m_0 c^{2.}$$

Jogaby: $$E = c\sqrt{m_0^2 c^2 + 4\pi^2 \left(\frac{\hbar}{\lambda}\right)^2} - m_0 c^{2.}$$

3.6. Elektronyň haýsy kinetik energiýasynda *de Broýl* tolkun uzynlygy Komptonyňka deň?

Çözülişi: Belli bolşy ýaly, Komptonyň tolkun uzynlygy

$$\lambda = \frac{2\pi\hbar}{m_0 c}$$

de Broýlyňky bolsa

$$\lambda = \frac{2\pi\hbar}{P}$$

bu ýerde $P = \frac{m_0 c\beta}{\sqrt{1-\beta^2}}$

Şu aňlatmalardan

$$\frac{2\pi\hbar}{m_0 c} = \frac{2\pi\hbar\sqrt{1-\beta^2}}{m_0 c\beta}$$

ýa – da

$$\beta = \sqrt{1-\beta^2}$$

Şu ýerden tapýarys:

$$\beta^2 = \frac{1}{2}$$

Kinetik energiýa

$$E_{kin} = m_0 c^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right] = m_0 c^2\left[\frac{1}{\sqrt{1-\frac{1}{2}}} - 1\right]$$

Gutarnykly görnüşde

$$E_{kin} = m_0 c^2\left(\sqrt{2} - 1\right) = 0{,}51\,MeW\ (1{,}4 - 1) = 0{,}212\ MeW$$

Jogaby: $E_{kin} = 0{,}212\ MeW$

3.7. Elektron atom ölçeginiň $(10^{-10}\,m)$ çäginde hereket edýär. Elektronyň alyp biljek minimal energiýasyny tapmaly.

Berlen:

$\Delta x \approx 10^{-10}\,m$

$\hbar = 1{,}05\cdot 10^{-34}\,J\cdot s$

$m = 9{,}1\cdot 10^{-31}\,kg$

$E - ?$

Çözüwi:

$\Delta x\cdot\Delta P \approx \hbar$ Kesgitsizlik gatnaşygyna layyklykda

bu ýerden

$$\Delta P = \frac{\hbar}{\Delta x} = \frac{1{,}05\cdot 10^{-34}\,J\cdot s}{10^{-10}\,m} = 1{,}05\cdot 10^{-24}\,\frac{kg\cdot m}{s}$$

Elektronyñ alyp biljek minimal energiýasy

$$E=\frac{(\Delta P)^2}{2m}=\frac{\left(1{,}05\cdot10^{-24}\frac{kg\cdot m}{s}\right)^2}{2\cdot9{,}1\cdot10^{-31}\,kg}=\frac{1{,}1\cdot10^{-17}}{18{,}2}J=0{,}06\cdot10^{-17}J=0{,}037\cdot10^{2}eW\approx3{,}7eW$$

Jogaby: $E\approx3{,}7eW$

3.8. Wodorod gazy, radiusy 0.01m bolan sfera görnüşli gapda ýerleşen. Wodorodyň molekulasynyň bolup biljek minimal energiýasyny tapmaly.

Berlen:

$\Delta x=0{,}01m$

$\hbar=1{,}05\cdot10^{-34}\,J\cdot s$

$E-?$

Çözüwi:

$$\Delta P=\frac{\hbar}{\Delta x}=\frac{1{,}05\cdot10^{-34}\,J\cdot s}{0{,}01m}=105\cdot10^{-34}\frac{kg\cdot m}{s}$$

$$E=\frac{(\Delta P)^2}{2m}=\frac{\left(105\cdot10^{-34}\frac{kg\cdot m}{s}\right)^2}{2\cdot2{,}35\cdot10^{-27}\,kg}=2346\cdot10^{-41}J=$$

$$=1466\cdot10^{-22}eW\approx1{,}5\cdot10^{-19}eW$$

Jogaby: $E\approx1{,}5\cdot10^{-19}eW$

3.9. Otag temperaturasynda ýylylyk hereketine gatnaşýan wodorodyň molekulasynyň $(m=2{,}35\cdot10^{-27}\,kg)$koordinatasy nähili takyklyk bilen tapylyp bilner?

Berlen:

$m=2{,}35\cdot10^{-27}\,kg$

$\hbar=1{,}05\cdot10^{-34}\,J\cdot s$

$T=300\ K$

$k=1{,}38\cdot10^{-34}\frac{J}{K}$

$\Delta x-?$

Çözüwi:

Otag temperaturasynda ýylylyk energiýasynyň deňölçegli paýlanma kanuny kanagatlandyrylýar.

Şonuň üçin ýylylyk hereketiniň orta energiýasy $\frac{3}{2}kT$ bolar,

ýagny $\frac{\overline{P^2}}{2m}=\frac{3}{2}kT$ we $\sqrt{\overline{P^2}}=\sqrt{3m\cdot kT}$

Islendik bölejigiň impulsynyň ortaça nätakyklygy

$$\overline{\Delta P} \sim \sqrt{\overline{P}^2} = \sqrt{3m \cdot kT}$$

Onuň koordinatasyny kesgitlemekdäki nätakyklyk:

$$\Delta x = \frac{\hbar}{\sqrt{3m \cdot kT}} = \frac{1{,}05 \cdot 10^{-34}\ J \cdot s}{\sqrt{3 \cdot 2{,}35 \cdot 10^{-27}\ kg \cdot 1{,}38 \cdot 10^{-23}\ \frac{J}{grad} \cdot 300\ grad}} \approx 10^{-11}\ m$$

Jogaby: $\Delta x \approx 10^{-11}\ m$

3.10. Wodorodyň atomynda üçinji Bor orbitasynda ýerleşýän elektron üçin de Broýlyň tolkun uzynlygyny kesgitlemeli.

Berlen:

$m = 9{,}1 \cdot 10^{-31}\ kg$

$h = 6{,}63 \cdot 10^{-34}\ J$

$n = 3$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ \frac{Kl^2}{J \cdot m}$

λ - ?

Çözüwi:

Elektron üçin hereket deňlemesini we impuls momentiniň kwantlanma düzgünini ýazalyň:

$$\frac{m\upsilon^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2},$$

$$m\upsilon r = n\hbar.$$

Şu deňlemelerden taparys:

$$\upsilon = \frac{e^2}{4\pi\varepsilon_0\hbar} \cdot \frac{1}{n} = \frac{e^2}{\varepsilon_0 h} \cdot \frac{1}{n}$$

Onda gözlenilýän ululyk:

$$\lambda = \frac{2\pi\hbar}{P} = \frac{h}{m\upsilon} = \frac{2h^2 n\varepsilon_0}{me^2}.$$

$$\lambda = \frac{2 \cdot \left(6{,}63 \cdot 10^{-34}\right)^2 \cdot 3 \cdot 8{,}85 \cdot 10^{-12}}{9{,}1 \cdot 10^{-31} \cdot \left(1{,}6 \cdot 10^{-19}\right)^2}\ m \approx 10^{-9}\ m = 1nm.$$

3.11. Oýandyrylan ýagdaýda atomyň ýaşamaklygynyň orta wagty $12\ ns$. Esasy ýagdaýa atomyň geçmegindäki şöhlelenmäniň $12\ mkm$ tolkun uzynlygynyň minimum kesgitsizligini hasaplamaly.

Berlen:

$\Delta t = 1{,}2 \cdot 10^{-8}\ s$

$\lambda = 1{,}2 \cdot 10^{-7}\ m$

$\Delta\lambda$ - ?

Çözüwi:

Şöhlelenýän fotonyň energiýasy

$$E = \frac{2\pi\hbar c}{\lambda}$$

Şuny λ boýunça differensirläliň:

$$dE = -2\pi\hbar c \frac{d\lambda}{\lambda^2}$$

ýa-da

$$\Delta E = -2\pi\hbar c \frac{\Delta\lambda}{\lambda^2}. \qquad \textbf{(1)}$$

Energiýa we wagt üçin Geýzenbergiň

$$\Delta E \cdot \Delta t \geq \hbar$$

kesgitsizlik gatnaşygyndan

$$\Delta E = \frac{\hbar}{\Delta t}. \qquad \textbf{(2)}$$

(1) we (2)-niň sag taraplaryny deňeşdireliň:

$$2\pi\hbar c \cdot \frac{\Delta\lambda}{\lambda^2} = \frac{\hbar}{\Delta t}$$

Şu ýerden

$$\Delta\lambda = \frac{\lambda^2}{2\pi c \cdot \Delta t}.$$

$$\Delta\lambda = \frac{1,44 \cdot 10^{-14}}{6,28 \cdot 3 \cdot 10^{8} \cdot 1,2 \cdot 10^{-8}} m \approx 6,4 \cdot 10^{-16} m.$$

IV bap

Kwant mehanikasynyň matematiki apparaty

IV.1. Usuly görkezmeler

- Kwant mehanikasynda operator düşünjesi giňden ulanylýar. Berlen "u" funksiýadan başga bir "ϑ" funksiýany almak üçin ulanylýan matematiki amala operator diýilýär. Ol şeýle belgilenýär:

$$\upsilon = \hat{L}U$$

- Kwant mehanikasynda dine çyzykly özüneçatrymly (ermit) operatory ulanylýar. Operatoryň çyzyklylygynyň şerti:

$$\hat{L}\left(c_1 U_1 + c_2 U_2\right) = c_1 \hat{L} U_1 + c_2 \hat{L} U_2$$

- Çyzykly operatoryň özüneçatrymlylygynyň şerti:

$$\int_{-\infty}^{+\infty} U_1^* \hat{L} U_2 dx = \int_{-\infty}^{+\infty} U_2 \hat{L}^* U_1^* dx$$

$\hat{L}$ we $\hat{M}$ *operatorlaryň kommutatory* diyip $\left(\hat{L}\hat{M} - \hat{M}\hat{L}\right)$ ululyga aýdylýar.

- Operatoryň hususy bahasynyň we hususy funksiýasynyň deňlemesi:

$$\hat{L}\psi = L\psi$$

- Kwant mehanikasynyň ýönekeý operatorlaryna koordinatanyň we impulsyň operatory, impulsyň momentiniň operatory we umumy energiýanyň $\hat{H}$ operatory (gamiltonian) girýär. Olar :

$$\hat{P}_x = -i\hbar \frac{\partial}{\partial x}; \qquad \hat{P}_y = -i\hbar \frac{\partial}{\partial y}; \qquad \hat{P}_z = -i\hbar \frac{\partial}{\partial z};$$

$$\hat{M}_x = y\hat{P}_z - z\hat{P}_y,$$

$$\hat{M}_y = z\hat{P}_x = x\hat{P}_z,$$

$$\hat{M}_z = x\hat{P}_y - y\hat{P}_x;$$

$$\hat{H} = \frac{1}{2\mu}\left(\hat{P}_x^2 + \hat{P}_y^2 + \hat{P}_z^2\right) + \hat{U}(x, y, z, t).$$

IV.2. Meseleler

4.1. Aşakdaky operatorly deňlemeleriň dogrulygyny barlamaly:

a) $\left(\frac{d}{dx}\cdot x\right)^2 = 1+3x\frac{d}{dx}+x^2\frac{d^2}{dx^2}$;

b) $\left(\frac{1}{x}\cdot\frac{d}{dx}\cdot x\right)^2 = \frac{d^2}{dx^2}+\frac{2}{x}\cdot\frac{d}{dx}$;

ç) $x^2\cdot\frac{d}{dx}\cdot\frac{1}{x} = x\cdot\frac{d}{dx}-1$;

d) $\left(\frac{d}{dx}+x\right)^2 = \frac{d^2}{dx^2}+2x\frac{d}{dx}+x^2+1$;

e) $\left(\frac{d}{dx}+\frac{1}{x}\right)^3 = \frac{d^3}{dx^3}+\frac{3}{x}\cdot\frac{d^2}{dx^2}$;

ä) $\left(\hat{L}-\hat{M}\right)\left(\hat{L}+\hat{M}\right)=\hat{L}^2-\hat{M}^2+\left(\hat{L}\hat{M}-\hat{M}\hat{L}\right)$.

Çözüwi:

a) Çep tarapdaky operatory erkin funksiýa täsir etdirýäris:

$$\left(\frac{d}{dx}\cdot x\right)^2\psi = \left(\frac{d}{dx}\cdot x\right)\cdot\left(\frac{d}{dx}\cdot x\right)\psi = \left(\frac{d}{dx}\cdot x\right)\cdot\frac{d}{dx}(x\psi) = \left(\frac{d}{dx}\cdot x\right)\cdot\left(\psi + x\cdot\frac{d\psi}{dx}\right) = \frac{d}{dx}(x\cdot\psi)+\frac{d}{dx}\left(x^2\cdot\frac{d\psi}{dx}\right) =$$

$$= \psi + x\cdot\frac{d\psi}{dx}+2x\cdot\frac{d\psi}{dx}+x^2\cdot\frac{d^2\psi}{dx^2} = \psi+3x\cdot\frac{d\psi}{dx}+x^2\cdot\frac{d^2\psi}{dx^2},$$

ψ -ni aýryp alýarys:

$$\left(\frac{d}{dx}\cdot x\right)^2 = 1+3x\frac{d}{dx}+x^2\frac{d^2}{dx^2}.$$

4.2. Operatorlaryň kommutatorlaryny tapmaly.

a) x we $\frac{d}{dx}$

b) $i\hbar\nabla$ we $\hat{A}(\vec{r})$.

Çözüwi:

a) x we $\frac{d}{dx}$ — operatorlaryň kommutatory $\left(x\frac{d}{dx}-\frac{d}{dx}x\right)$. Şu operatory ψ funksiýa täsir etdireliň:

$$\left(x\frac{d}{dx}-\frac{d}{dx}x\right)\cdot\psi = x\frac{d\psi}{dx}-\frac{d}{dx}(x\cdot\psi) = x\frac{d\psi}{dx}-\psi-x\frac{d\psi}{dx} = -\psi\ ;$$

ψ -ni aýryp alarys:

$$x\frac{d}{dx}-\frac{d}{dx}x = -1.$$

b) $\left(i\hbar\nabla\cdot\hat{A}-\hat{A}\cdot i\hbar\nabla\right)\cdot\psi = i\hbar\nabla\left(\hat{A}\cdot\psi\right)-i\hbar\hat{A}\nabla\cdot\psi = i\hbar\nabla\hat{A}\cdot\psi+i\hbar\hat{A}\nabla\cdot\psi-i\hbar\hat{A}\nabla\cdot\psi = i\hbar\nabla\hat{A}\cdot\psi$

ψ -ni ayryp:

$$i\hbar\left(\nabla\cdot\hat{A}-\hat{A}\cdot\nabla\right) = i\hbar\hat{A}\nabla = i\hbar\cdot div\,\hat{A}$$

4.3. $\frac{d^2}{dx^2}\cdot x^2$ *we* $\left(\frac{d}{dx}\cdot x\right)^2$ operatorlaryň *a)* $\sin x$, *b)* $\cos x$ we *ç)* e^{2x} funksiýalara täsirleriniň netijesini tapmaly.

Çözüwi:

a) $$\left(\frac{d^2}{dx^2}\cdot x\right)\sin x = \frac{d}{dx}\cdot\frac{d}{dx}\left(x^2\cdot\sin x\right) = \frac{d}{dx}\left(2x\cdot\sin x+x^2\cos x\right) = \frac{d}{dx}\left(2x\cdot\sin x\right)+\frac{d}{dx}\left(x^2\cdot\cos x\right) =$$
$$= 2\sin x+2x\cos x+2x\cos x-x^2\sin x = 2\sin x+4x\cos x-x^2\sin x$$

we

$$\left(\frac{d}{dx}\cdot x\right)^2\sin x = \left(\frac{d}{dx}\cdot x\right)\cdot\left(\frac{d}{dx}\cdot x\right)\cdot\sin x = \left(\frac{d}{dx}\cdot x\right)\cdot\frac{d}{dx}\cdot(x\cdot\sin x) = \left(\frac{d}{dx}\cdot x\right)\cdot(\sin x+x\cdot\cos x) =$$

$$= \frac{d}{dx}(x \cdot \sin x) + \frac{d}{dx} \cdot (x^2 \cdot \cos x) = \sin x + x \cdot \cos x + 2x \cdot \cos x - x^2 \cdot \sin x = \sin x + 3x \cdot \cos x - x^2 \cdot \sin x \text{ .}$$

4.4. $\hat{L}\hat{M} - \hat{M}\hat{L} = 1$ gatnaşygy kanagatlandyrýan $\hat{L}$ we $\hat{M}$ operatorlar üçin $\hat{L}\hat{M}^2 - \hat{M}^2\hat{L}$ gatnaşygy tapmaly.

4.5. $U(x)$ potensial meýdanda $\hat{H}$ gamiltonian üçin aşakdaky kommutasiýa gatnaşyklaryny barlamaly.

a) $\left[\hat{H}\hat{X}\right] = -\frac{i\hbar}{\mu}\hat{P}_x$ **;** **b)** $\left[\hat{H}\hat{P}_x\right] = i\hbar\frac{\partial U}{\partial x}$ **;** **ç)** $\left[\hat{H}\hat{P}_x^2\right] = 2i\hbar\frac{\partial U}{\partial x}\hat{P}_x + \hbar^2\frac{\partial^2 U}{\partial x^2}$ **.**

4.6. $\hat{P} = -ie^{ix}\frac{d}{dx}$ operatoryň hususy funksiýasyny tapmaly.

Çözüwi:
Kesgitleme boýunça:

$$\hat{P}\psi = \lambda\psi \ ,$$

bu ýerde λ - hususy baha.

$$-ie^{ix}\frac{d\psi}{dx} = \lambda\psi \ ,$$

ýa-da

$$\frac{d\psi}{\psi} = i\lambda e^{-ix}dx \text{ .}$$

Şu aňlatmany integrirläp we soňra potensirläp, taparys:

$$\psi = \exp\left(i\lambda e^{-ix}\right)$$

4.7. $\hat{P} + \hat{X}$ operatoryň birölçegli hereketde hususy funksiýasyny we hususy bahasyny tapmaly.

Çözülişi:
Hususy deňlemäni ýazýarys:

$$\left(\hat{P} + \hat{X}\right) \cdot \psi = \lambda\psi \ ,$$

ýa-da

$$\left(\frac{\hbar}{i}\cdot\frac{\partial}{\partial x}+x\right)\cdot\psi = \lambda\psi \ ,$$

sebäbi

$$\hat{P}_x = \frac{\hbar}{i}\cdot\frac{\partial}{\partial x}$$

Onda

$$\frac{\hbar}{i}\cdot\frac{\partial\psi}{\partial x} = \lambda\psi - x\cdot\psi$$

ýa – da

$$\frac{\hbar}{i}\cdot\frac{\partial\psi}{\partial x} = (\lambda - x)\cdot\psi$$

Şu aňlatmany integrirleýäris:

$$\frac{\hbar}{i}\cdot\ln\psi = \left(\lambda x - \frac{x^2}{2}\right) + \ln c, \ ;$$

Indi potensirleýäris:

$$\psi = const\cdot e^{\frac{i}{\hbar}\left(\lambda x - \frac{x^2}{2}\right)} .$$

Şu ýerden görnüşi ýaly λ – nyň ähli hakyky bahalarynda ψ gutarnykly bahalary alýar.

4.8. Impulsyň operatorynyň hususy funksiýalaryny tapmaly.

Çözüwi:

$\frac{\hbar}{i}\cdot\frac{\partial\psi}{\partial x} = P_x\cdot\psi$; $\frac{\hbar}{i}\cdot\frac{\partial\psi}{\partial y} = P_y\cdot\psi$; $\frac{\hbar}{i}\cdot\frac{\partial\psi}{\partial z} = P_z\cdot\psi$ ýazyp bileris.

Bu ýerde P_x, P_y, P_z - impulsyň wektorynyň düzüjileriniň operatorlarynyň hususy bahalary.

Deňlemeleriň çözgüdi:

$$\psi = e^{\frac{i}{\hbar}(P_x x + P_y y + P_z z)} ,$$

bu öz gezeginde impulsyň düzüjileriniň operatorynyň hususy funksiýasy bolup hyzmat edýär.

4.9. Aşakdaky operatorlaryň hususy funksiýalaryny we hususy bahalaryny tapmaly:

a) $\frac{d}{dx}$**;** **b)** $i\cdot\frac{d}{dx}$; **ç)** $x+\frac{d}{dx}$;

4.10. $\hat{P}_x$**,** $\hat{P}_y$**,** $\hat{P}_z$ operatorlaryň we Laplasyň operatorynyň ermitligini barlamaly.

4.11. Impulsyň momentiniň $\hat{M}_z$ proýeksiýasynyň ermitligini subut etmeli:

$$\hat{M}_z = -i\hbar\cdot\frac{\partial}{\partial\varphi} \quad ,$$

Bu ýerde $0\le\varphi\le 2\pi$ **.**

Çözülişi:

$$\int_{-\infty}^{+\infty}\psi_1^*\cdot\hat{L}\psi_2 d\nu = \int_{-\infty}^{+\infty}\psi_2\cdot\hat{L}^*\cdot\psi_1^* d\nu \cdot$$

şerti kanagatlandyrýan operatora özüneçatrymly ýa-da ermitli operator diýilýär.
Şu mesele üçin:

$$\int_0^{2\pi}\psi_1^*\hat{M}_z\psi_2 d\varphi = -i\hbar\int_0^{2\pi}\psi_1^*\frac{\partial}{\partial\varphi}\psi_2\cdot d\varphi = -i\hbar\left(\psi_1^*\cdot\psi_2\right)\Big|_0^{2\pi} - i\hbar\int_0^{2\pi}\psi_2\frac{\partial\psi_1^*}{\partial\varphi}\cdot d\varphi = -i\hbar\left(\psi_1^*\cdot\psi_2\right)\Big|_0^{2\pi} +$$

$$+\int_0^{2\pi}\psi_2\hat{M}_z^*\psi_1^*\cdot d\varphi \;.$$

ψ^* we ψ_2 funksiýalar tebigy şertleri meselem, birbahalylygy kanagatlandyrýarlar, onda

$$\left(\psi_1^*\psi_2\right)\Big|_{\varphi=0} = \left(\psi_1^*\psi_2\right)\Big|_{\varphi=2\pi} \;,$$

ýagny

$$\left(\psi_1^*\psi_2\right)\Big|_0^{2\pi} = 0$$

Diýmek:

$$\int_0^{2\pi}\psi_1^*\hat{M}_z\psi_2 d\varphi = \int_0^{2\pi}\psi_2\hat{M}_z^*\psi_1^* d\varphi \;,$$

ýagny

$\hat{M}_z = -i\hbar\frac{\partial}{\partial\varphi}$ operatory ermitdir.

4.12. Aşakdaky operatorlaryň ermitligini subut etmeli.

a) $\hat{P}_x$; **b)** $\hat{P}_x^2$; **ç)** $\hat{H}$ **.**

Çözüwi:

ç) $\hat{H} = \dfrac{\hat{P}^2}{2\mu} + \hat{U} = -\dfrac{\hbar^2}{2\mu}\Delta + \hat{U}$ **.**

$$\int_{-\infty}^{+\infty} \psi_1^* \Delta \psi_2 dx = -\psi_1^* grad\ \psi_2 \Big|_{-\infty}^{+\infty} + \int grad\ \psi_2 \cdot grad\ \psi_1^* dx = \psi_2 grad\ \psi_1^* \Big|_{-\infty}^{+\infty} - \psi_2 \int_{-\infty}^{+\infty} \psi_2 \Delta \psi_1^* dx =$$

$$= \int_{-\infty}^{+\infty} (\Delta \psi_1)^* \psi_2 dx\ ,$$

we $\quad \int \psi_1^* U \psi_2 dx = \int_{-\infty}^{+\infty} (\psi_1 U)^* \psi_2 dx$

4.13. $\hat{P} = -ie^{ix}\dfrac{d}{dx}$ we $\overleftarrow{Q} = e^{ix}$ operatorlaryň kommutasiýa düzgünini tapmaly.

Çözüwi:

$$\left(\hat{P}\hat{Q} - \hat{Q}\hat{P}\right)\cdot\psi = -ie^{ix}\frac{d}{dx}\cdot e^{ix}\psi + ie^{ix}\cdot e^{ix}\frac{d\psi}{dx} = -ie^{ix}\cdot ie^{ix}\psi - ie^{ix}\cdot ie^{ix}\frac{d\psi}{dx} + ie^{ix}\cdot ie^{ix}\frac{d\psi}{dx} = e^{2ix}\psi .$$

ψ - ni aýyrýarys:

$$\hat{P}\hat{Q} - \hat{Q}\hat{P} = e^{2ix}.$$

4.14. $\hat{P}_y$ we erkin $f(y)$ funksiýanyň kommutasiýa düzgünini tapmaly.

4.15. Sistemanyň tolkun funksiýasy aşakdaky görnüşde berilýär.

$$\psi = \varphi(x)\cdot e^{-\frac{i}{\hbar}Et} + \varphi(x)\cdot e^{\frac{i}{\hbar}Et}$$

Sistemanyň ähtimallygynyň paýlanmasyny tapmaly we onuň stasionar ýagdaýda däldigini esaslandyrmaly.

Çözüwi:

Ýagdaýyň ähtimallygy

$$\psi\psi^{*} = \left[\varphi(x)\cdot e^{-\frac{i}{\hbar}Et} + \varphi(x)\cdot e^{\frac{i}{\hbar}Et}\right]\left[\varphi(x)\cdot e^{\frac{i}{\hbar}Et} + \varphi(x)\cdot e^{-\frac{i}{\hbar}Et}\right] = \varphi^{2}(x)\left[e^{\frac{i}{\hbar}Et} + e^{-\frac{i}{\hbar}Et}\right]^{2} =$$

$$= \varphi^{2}(x)\left(2 + e^{\frac{2i}{\hbar}Et} + e^{-\frac{2i}{\hbar}Et}\right) = 2\varphi^{2}(x)\cdot\left[1 + \cos\frac{2E}{\hbar}t\right].$$

deňdir:

Görnüşi ýaly, ähtimallygyň paýlanmasy wagta bagly, ýagny ulgamyň ýagdaýy stasionar däl.

4.16. Wagta aýdyň bagly bolan operatoryň wagta görä önümi üçin aňlatmany tapmaly.

Çözüwi:

$$\overline{F} = \int \psi^{*}\hat{F}\psi\cdot dv$$ ýazyp bileris.

Onuň wagta görä önümi

$$\frac{d\overline{F}}{dt} = \int\frac{\partial\psi^{*}}{\partial t}\hat{F}\psi\cdot dv + \int\psi^{*}\frac{\partial\hat{F}}{\partial t}\psi\cdot dv + \int\psi^{*}\hat{F}\frac{\partial\psi}{\partial t}\cdot dv$$

Birinji we üçünji integrallary özgerdip, alarys:

$$\int\frac{\partial\psi^{*}}{\partial t}\hat{F}\psi\cdot dv + \int\psi^{*}\hat{F}\frac{\partial\psi}{\partial t}\cdot dv = \int\psi^{*}\left[\hat{H}\hat{F}\right]\cdot\psi dv;$$

Şonuň üçin

$$\frac{d\overline{F}}{dt} = \int\psi^{*}\left\{\left[\hat{H}\hat{F}\right] + \frac{\partial\hat{F}}{\partial t}\right\}\psi\cdot dv;$$

Şu ýerden taparys:

$$\frac{d\hat{F}}{dt} = \frac{\partial \hat{F}}{\partial t} + \left[\hat{H}\hat{F}\right].$$

4.17. Birjynsly magnit meýdanda $\hat{P}$ we $\hat{A}$ operatorlar üçin kommutasiýa düzgünini tapmaly.

Çözüwi:

Kommutatory emele getirýäris we ψ – ä täsir etdirýäris:

$$\left(\hat{P}\hat{A} - \hat{A}\hat{P}\right)\psi = \hat{P}\hat{A} - \hat{A}_x\hat{P}_x - \hat{A}_y\hat{P}_y - \hat{A}_z\hat{P}_z = \left(\hat{P}\hat{A} - \hat{P}_x\hat{A}_x - i\hbar\frac{\partial \hat{A}_x}{\partial x} - \hat{P}_y\hat{A}_y - i\hbar\frac{\partial \hat{A}_y}{\partial y} - \hat{P}_z\hat{A}_z - i\hbar\frac{\partial \hat{A}_z}{\partial z}\right)\psi =$$

$$\left\{\hat{P}_x\hat{A}_x \pm \hat{P}_y\hat{A}_y \pm \hat{P}_z\hat{A}_z - \hat{P}_x\hat{A}_x - \hat{P}_y\hat{A}_y - \hat{P}_z\hat{A}_z - i\hbar\left(\frac{\partial \hat{A}_x}{\partial x} + \frac{\partial \hat{A}_y}{\partial y} + \frac{\partial \hat{A}_z}{\partial z}\right)\psi\right\} = \frac{\hbar}{i}\psi divA .$$

Şeýlelikde , ψ – ni aýryp alarys:

$$\hat{P}\hat{A} - \hat{A}\hat{P} = \frac{\hbar}{i} div\,\hat{A}.$$

4.18. Magnit meýdanda impulsyň proýeksiýalary üçin kommutasiýa düzgünini tapmaly.

Çözüwi:

Kommutatory düzýäris we ψ – ä täsir etdirýäris:

$$\left\{\left(\hat{P}_x - \frac{e}{c}\hat{A}_x\right)\left(\hat{P}_y - \frac{e}{c}\hat{A}_y\right) - \left(\hat{P}_y - \frac{e}{c}\hat{A}_y\right)\left(\hat{P}_x - \frac{e}{c}\hat{A}_x\right)\right\}\psi =$$

$$= \left(\hat{P}_x\hat{P}_y - \frac{e}{c}\hat{P}_x\hat{A}_y - \frac{e}{c}\hat{A}_x\hat{P}_y + \frac{e^2}{c^2}\hat{A}_x\hat{A}_y - \hat{P}_y\hat{P}_x + \frac{e}{c}\hat{P}_y\hat{A}_x\,\frac{e}{c}\hat{A}_y\hat{P}_x - \frac{e^2}{c^2}\hat{A}_y\hat{A}_x\right)\psi =$$

$$= \frac{e}{c}\left\{\left(\hat{P}_y\hat{A}_x - \hat{A}_x\hat{P}_y\right) - \left(\hat{P}_x\hat{A}_y - \hat{A}_y\hat{P}_x\right)\right\}\psi = \frac{e}{c}\left(-i\hbar\frac{\partial \hat{A}_x}{\partial y} + i\hbar\frac{\partial \hat{A}_y}{\partial x}\right)\psi = -\frac{i\hbar e}{c}\left(\frac{\partial \hat{A}_y}{\partial x} - \frac{\partial \hat{A}_x}{\partial y}\right)\psi = \frac{\hbar e}{ic}\hat{\mathscr{H}}_z\psi$$

ψ – ni aýryp tapýarys:

$$\left(\hat{P}_x - \frac{e}{c}\hat{A}_x\right)\left(\hat{P}_y - \frac{e}{c}\hat{A}_y\right) - \left(\hat{P}_y - \frac{e}{c}\hat{A}_y\right)\left(\hat{P}_x - \frac{e}{c}\hat{A}_x\right) = \frac{\hbar e}{ic}\hat{\mathscr{H}}_z$$

4.19. Aşakdaky gatnaşyklaryň ýerine ýetýändiklerini subut etmeli:

a)
$$\left.\begin{aligned} y\hat{P}_y - \hat{P}_y y &= i\hbar \\ z\hat{P}_z - \hat{P}_z z &= i\hbar. \end{aligned}\right\}$$

b)
$$\left.\begin{aligned} y\hat{P}_x - \hat{P}_x y &= 0, \\ y\hat{P}_z - \hat{P}_z y &= 0. \end{aligned}\right\}$$ we ş.m.

c)
$$\left.\begin{aligned} y\hat{P}_y^2 - \hat{P}_y^2\, y &= 2i\hbar \cdot \hat{P}_y, \\ z\hat{P}_z^2 - \hat{P}_z^2 z &= 2i\hbar \cdot \hat{P}_Z. \end{aligned}\right\}$$

we
$$\left.\begin{aligned} x^2\hat{P}_x - \hat{P}_x x^2 &= 2i\hbar \cdot x, \\ y^2\hat{P}_y - \hat{P}_y y^2 &= 2i\hbar \cdot y, \\ z^2\hat{P}_z - \hat{P}_z z^2 &= 2i\hbar \cdot z. \end{aligned}\right\}$$

d)
$$\left.\begin{aligned} \hat{M}_y\hat{M}_z - \hat{M}_z\hat{M}_y &= i\hbar \cdot \hat{M}_x, \\ \hat{M}_z\hat{M}_x - \hat{M}_x\hat{M}_z &= i\hbar \cdot \hat{M}_y, \\ \hat{M}_y\hat{M}^2 - \hat{M}^2\hat{M}_y &= 0, \\ \hat{M}_z\hat{M}^2 - \hat{M}^2\hat{M}_z &= 0. \end{aligned}\right\}$$

4.20. Impulsyň momentiň düzüjisiniň operatorynyň we koordinata operatorlaryň arasyndaky kommutasiýalary tapmaly.

Çözüwi:

Kommutatorlary emele getirip we özgerdip, taparys:

$$\hat{M}_x x - x\hat{M}_x = 0,$$
$$\hat{M}_x y - y\hat{M}_x = i\hbar z,$$
$$\hat{M}_x z - z\hat{M}_x = -i\hbar y,$$
$$\hat{M}_y z - z\hat{M}_y = i\hbar x.$$

4.21. Impulsyň operatorynyň düzüjileriniň we $\hat{M}_z$ operatory bilen ýerine ýetýän kommutasiýa düzgünini tapmaly.

Çözüwi:

1. $\hat{P}_x\hat{M}_z - \hat{M}_z\hat{P}_x = \hat{P}_x\left(x\hat{P}_y - y\hat{P}_x\right) - \left(x\hat{P}_y - y\hat{P}_x\right)\hat{P}_x = \hat{P}_x x\hat{P}_y - \hat{P}_x y\hat{P}_x - x\hat{P}_y\hat{P}_x + y\hat{P}_x\hat{P}_x =$

$= \hat{P}_y\left(\hat{P}_x x - x\hat{P}_x\right) = -i\hbar\hat{P}_y = \dfrac{\hbar}{i}\hat{P}_y.$

Diýmek,

$$\hat{P}_x\hat{M}_z - \hat{M}_z\hat{P}_x = \frac{\hbar}{i}\hat{P}_y.$$

2. $\hat{P}_y\hat{M}_z - \hat{M}_z\hat{P}_y = i\hbar\hat{P}x$

3. $\hat{P}_z\hat{M}_z - \hat{M}_z\hat{P}_z = 0$

4.22. Aşakdaky operator deňlemeleri subut etmeli.

a) $y\hat{P}_y^2 - \hat{P}_y^2 y = 2i\hbar\hat{P}_y,$

b) $z^2\hat{P}_z - \hat{P}_z z^2 = 2i\hbar.$

4.23. $U(x)$ potensial meýdanda $\hat{H}$ gamiltonian üçin aşakdaky kommutasiýa gatnaşyklary barlamaly.

a) $\left[\hat{H}x\right] = \dfrac{P_x}{\mu};$ **b)** $\left[\hat{H}\hat{P}_x\right] = -\dfrac{\partial U}{\partial x};$ **ç)** $\left[\hat{H}\hat{P}_x^2\right] = -2\dfrac{\partial U}{\partial x}\hat{P}_x + i\hbar\dfrac{\partial^2 U}{\partial x^2}.$

Çözüwi:

ç) Erkin bölejik üçin gamiltonian:

$$\hat{H} = \frac{\hat{P}_x^2}{2\mu} + U(x);$$

Onda

$$\left[\hat{H}\hat{P}_x^2\right] = \frac{1}{i\hbar}\left(\hat{P}_x^2\hat{H} - \hat{H}\hat{P}_x^2\right) = \frac{1}{i\hbar}\left(\hat{P}_x^2 U - U\hat{P}_x^2\right) = \frac{1}{i\hbar}\left(\hat{P}_x^2 U - U\hat{P}_x\cdot\hat{P}_x\right) = \frac{1}{i\hbar}\left\{\hat{P}_x^2 U - \left(\hat{P}_x U + i\hbar\frac{\partial U}{\partial x}\right)\cdot\hat{P}_x\right\} =$$

$$= \frac{1}{i\hbar}\left(\hat{P}_x^2 U - \hat{P}_x\cdot U\hat{P}_x - i\hbar\frac{\partial U}{\partial x}\hat{P}_x\right) = \frac{1}{i\hbar}\left\{\hat{P}_x^2 U - \hat{P}_x\left(\hat{P}_x U + i\hbar\frac{\partial U}{\partial x}\right) - i\hbar\frac{\partial U}{\partial x}\hat{P}_x\right\} =$$

$$= \frac{1}{i\hbar}\left(\hat{P}_x^2 U - \hat{P}_x\hat{P}_x U - i\hbar\hat{P}_x\frac{\partial U}{\partial x} - i\hbar\frac{\partial U}{\partial x}\hat{P}_x + i\hbar\frac{\partial U}{\partial x}\hat{P}_x - i\hbar\frac{\partial U}{\partial x}\hat{P}_x\right) =$$

$$= \frac{1}{i\hbar}\left\{-2i\hbar\frac{\partial U}{\partial x}\hat{P}_x + i\hbar\left(\frac{\partial U}{\partial x}P_x - \hat{P}_x\frac{\partial U}{\partial x}\right)\right\} = \frac{1}{i\hbar}\left(-2i\hbar\frac{\partial U}{\partial x}\hat{P}_x + (i\hbar)^2\frac{\partial}{\partial x}\frac{\partial U}{\partial x}\right) = -2\frac{\partial U}{\partial x}\cdot\hat{P}_x + i\hbar\frac{\partial^2 U}{\partial x^2}.$$

Diýmek, $\left[\hat{H}\hat{P}_x^2\right] = -2\frac{\partial U}{\partial x}\hat{P}_x + i\hbar\frac{\partial^2 U}{\partial x^2}$

4.24. Merkezi simmetrik meýdanda elektronyň deňölçegli hereketinde $\hat{M}^2$ we $\hat{M}_z$ operatorlaryň saklanýandygyny subut etmeli.

Çözüwi:

Impulsyň momentiň kwadratynyň operatorynyň wagta görä üýtgemegi:

$$\frac{d\hat{M}^2}{dt} = \left[\hat{H}\hat{M}\right]^2 \quad \text{deňdir.}$$

Ýöne $\hat{H}$ we $\hat{M}^2$ operatorlary kommutirleşýär, ýagny

$$\left[\hat{H}\hat{M}^2\right] = 0 \quad \text{we} \quad \frac{d\hat{M}^2}{dt} = 0$$

Edil şunuň ýaly

$$\frac{d\hat{M}_z}{dt} = \left[\hat{H}\hat{M}_z\right] = 0$$

4.25. $\hat{M}_x$, $\hat{M}_y$, $\hat{M}_z$ we $\hat{M}^2$ operatorlaryň bahalaryny dekart ulgamyndan sferiki ulgama özgertmeli.

Çözüwi:

$\hat{M}_x$ operatoryň bahasyny aşakdaky görnüşde ýazalyň:

$$\hat{M}_x = y\hat{P}_z - z\hat{P}_y = i\hbar\left(z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}\right) = i\hbar\left\{z\left(\frac{\partial r}{\partial y}\frac{\partial}{\partial r} + \frac{\partial\theta}{\partial y}\frac{\partial}{\partial\theta} + \frac{\partial\varphi}{\partial y}\frac{\partial}{\partial y}\right) - y\left(\frac{\partial r}{\partial z}\frac{\partial}{\partial r} + \frac{\partial\theta}{\partial z}\frac{\partial}{\partial\theta} + \frac{\partial\varphi}{\partial z}\frac{\partial}{\partial\varphi}\right)\right\} \quad \textbf{(1)}$$

we

$$x = r\sin\theta\cdot\cos\varphi, \qquad y = r\sin\theta\cdot\sin\varphi, \qquad z = r\cos\theta. \quad \textbf{(2)}$$

(2) – den:

$$tg\,\varphi = \frac{y}{x}, \quad \cos\theta = \frac{z}{x}, \quad \text{bulardan başga} \quad r^2 = x^2 + y^2 + z^2. \quad \textbf{(3)}$$

(3) – den aşakdaky önümleri hasaplalyň:

$$\frac{\partial r}{\partial y} = \sin\theta\cdot\sin\varphi, \quad \frac{\partial\theta}{\partial y} = \frac{\cos\theta\cdot\sin\varphi}{r}, \quad \frac{\partial\varphi}{\partial y} = \frac{\cos\varphi}{r\sin\theta}, \quad \frac{\partial r}{\partial z} = \cos\theta, \quad \frac{\partial\theta}{\partial z} = -\frac{\sin\theta}{r}, \quad \frac{\partial\varphi}{\partial z} = 0.$$

Şu önümleri we (2) - ni (1) - e goýup, alýarys:

$$\hat{M}_x = i\hbar\left\{r\cos\theta\left(\sin\theta\cdot\sin\varphi\frac{\partial}{\partial r} + \frac{\cos\theta\cdot\sin\varphi}{r}\frac{\partial}{\partial\theta} + \frac{\cos\varphi}{r\sin\theta}\frac{\partial}{\partial\varphi}\right) - r\sin\theta\cdot\sin\varphi\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\right\} =$$

$$= i\hbar\left(r\cos\theta\cdot\sin\theta\cdot\sin\varphi\frac{\partial}{\partial r} + \cos^2\theta\cdot\sin\varphi\frac{\partial}{\partial\theta} + ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi} - r\sin\theta\cdot\sin\varphi\cdot\cos\theta\frac{\partial}{\partial r} +\right.$$

$$\left.+\sin^2\theta\cdot\sin\varphi\frac{\partial}{\partial\theta}\right) = i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta} + ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi}\right).$$

Diýmek,

$$\hat{M}_x = i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta} + ctg\,\theta\cos\varphi\frac{\partial}{\partial\varphi}\right).$$

Edil şunuň ýaly

$$\hat{M}_y = -i\hbar\left(\cos\varphi\frac{\partial}{\partial\theta} - ctg\,\theta\cdot\sin\varphi\frac{\partial}{\partial\varphi}\right), \qquad \hat{M}_z = -i\hbar\frac{\partial}{\partial\varphi}.$$

Indi $\hat{M}^2 - i$ özgerdeliň:

$$\hat{M}^2 = \hat{M}_x^2 + \hat{M}_y^2 + \hat{M}_z^2 = \hat{M}_x\hat{M}_x + \hat{M}_y\hat{M}_y + \hat{M}_z\hat{M}_z = i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta} + ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi}\right)\cdot$$

$$\cdot i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta} + ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi}\right) + \left[-i\hbar\left(\cos\varphi\frac{\partial}{\partial\theta} - ctg\,\theta\cdot\sin\varphi\frac{\partial}{\partial\varphi}\right)\right]\cdot\left[-i\hbar\left(\cos\varphi\frac{\partial}{\partial\theta} - ctg\,\theta\cdot\sin\varphi\frac{\partial}{\partial\varphi}\right)\right] +$$

$$+\left(-i\hbar\frac{\partial}{\partial\varphi}\right)\cdot\left(-i\hbar\frac{\partial}{\partial\varphi}\right) = -\hbar^2\left(\sin^2\varphi\frac{\partial^2}{\partial\theta^2} + \sin\varphi\frac{\partial}{\partial\theta}ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi} + ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi}\sin\varphi\frac{\partial}{\partial\varphi} +\right.$$

$$+ ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi}ctg\,\theta\cdot\cos\varphi\frac{\partial}{\partial\varphi} + \cos^2\varphi\frac{\partial^2}{\partial\theta^2} - \cos\varphi\frac{\partial}{\partial\theta}ctg\,\theta\cdot\sin\varphi\frac{\partial}{\partial\varphi} - ctg\,\theta\cdot\sin\varphi\frac{\partial}{\partial\varphi}\cos\varphi\frac{\partial}{\partial\theta} +$$

$$\left.+ ctg\,\theta\cdot\sin\varphi\frac{\partial}{\partial\varphi}ctg\,\theta\cdot\sin\varphi\frac{\partial}{\partial\varphi} + \frac{\partial^2}{\partial\varphi^2}\right) = -\hbar^2\left(\frac{\partial^2}{\partial\theta^2} + ctg\,\theta\cdot\cos^2\frac{\partial}{\partial\theta} - ctg^2\theta\cdot\cos\varphi\cdot\sin\varphi\frac{\partial}{\partial\varphi} +\right.$$

$$\left.+ ctg^2\theta\cdot\cos^2\varphi\frac{\partial^2}{\partial\varphi^2} + ctg\,\theta\cdot\sin^2\varphi\frac{\partial}{\partial\theta} + ctg^2\theta\cdot\sin\varphi\cdot\cos\varphi\frac{\partial}{\partial\varphi} + ctg^2\theta\cdot\sin^2\varphi\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial\varphi^2}\right) =$$

$$= -\hbar^2\left[\left(\frac{\partial^2}{\partial\theta^2} + ctg\,\theta\frac{\partial}{\partial\theta}\right) + \left(ctg^2\theta + 1\right)\frac{\partial^2}{\partial\varphi^2}\right] = -\hbar^2\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right\},$$

ýa – da

$$\hat{M}^2 = -\hbar^2\nabla^2_{\theta,\varphi}.$$

4.26. Bölejigiň erkin hereketi bilen tolkun paketiniň ýaýramasynyň özara ekwiwalentdigini subut etmeli.

Çözüwi:

$\overline{(\Delta x)^2}$ orta kwadratik gyşarma $\Delta x^2 = x^2 - \bar{x}^2$ **(1)**

ululygyň orta bahasydyr, $\bar{x}$ – agyrlyk merkeziniň koordinatasynyň orta bahasy.

Onda $\frac{d\bar{x}}{dt} = \bar{v}$ ýa – da $\bar{x} = \bar{v}t + \overline{x_0}$ **(2)**

ýazyp bileris:

(2) – den görnüşi ýaly, paketiň merkezi $\bar{v}$ tizlikli hereket edýär.

(1)-iň wagta görä önümi şeýle tapylýar:

$$\frac{d(\Delta x)^2}{dt} = \frac{\partial(\Delta x)^2}{\partial t} + \left[\hat{H}(\Delta x)^2\right] \tag{3}$$

Ýöne

$$\left[\hat{H}(\Delta x)^2\right] = \left[\hat{H}\cdot x^2 - \bar{x}^2\right] = \left[\hat{H}\cdot x^2\right] - \left[\hat{H}\cdot \bar{x}^2\right] = \frac{dx^2}{dt} - \frac{d\bar{x}^2}{dt} \tag{4}$$

Erkin hereket üçin

$$\hat{H} = \frac{1}{2\mu} + \hat{P}^2,$$

onda

$$\left[\hat{H}\cdot x^2\right] = \frac{1}{i\hbar}\left(x^2\hat{H} - \hat{H}x^2\right) = \frac{1}{2\mu i\hbar}\left(x^2\hat{P}_x^2 - \hat{P}_x^2 x^2\right) = \frac{1}{2\mu i\hbar}\left\{x^2\hat{P}_x^2 - \hat{P}_x\left(x\hat{P}_x - i\hbar\right)\cdot x\right\} =$$

$$= \frac{1}{2\mu i\hbar}\left\{x^2\hat{P}_x^2 - \left(x\hat{P}_x - i\hbar\right)\cdot\hat{P}_x^2 x + i\hbar\hat{P}_x x\right\} = \frac{1}{2\mu i\hbar}\left\{x^2\hat{P}_x^2 - x\hat{P}_x\left(x\hat{P}_x - i\hbar\right) + 2i\hbar\hat{P}_x x\right\} =$$

$$= \frac{1}{2\mu i\hbar}\left\{x^2\hat{P}_x^2 - x\left(x\hat{P}_x - i\hbar\right)\cdot\hat{P}_x + i\hbar x\hat{P}_x + 2i\hbar\hat{P}_x x\right\} = \frac{1}{2\mu i\hbar}\left\{x^2\hat{P}_x^2 - xx\hat{P}_x\hat{P}_x + 2i\hbar x\hat{P}_x + 2i\hbar\hat{P}_x x\right\} = \frac{x\hat{P}_x + \hat{P}_x x}{\mu};$$

Şeýlelikde,

$$\frac{d(\Delta x)^2}{dt} = \frac{\partial(\Delta x)^2}{\partial t} + \frac{x\hat{P}_x + \hat{P}_x x}{\mu} - \frac{d\bar{x}^2}{dt} = \frac{x\hat{P}_x + \hat{P}_x x}{\mu} - \bar{v}\cdot\bar{x}. \tag{5}$$

sebäbi $\frac{d\bar{x}^2}{dt} = \frac{d\bar{x}}{dt}\cdot\bar{x} = \bar{v}\cdot\bar{x}$ we wagta bagly däl ýagdaýda $\frac{\partial(\Delta x)^2}{\partial t} = 0$

Indi ikinji önümi hasaplalyň:

$$\frac{d^2(\Delta x)^2}{dt^2} = \frac{d}{dt}\frac{d(\Delta x)^2}{dt} = \left[\hat{H}\;\frac{d(\Delta x)^2}{dt}\right] \tag{6}$$

Ýöne

$$\left[\hat{H}\;\frac{d(\Delta x)^2}{dt}\right] = \left[\hat{H}\left(\frac{x\hat{P}_x + \hat{P}_x x}{\mu} - \bar{v}\cdot\bar{x}\right)\right] = \frac{1}{\mu}\left[\hat{H}\left(x\hat{P}_x + \hat{P}_x x\right)\right] - \left[\hat{H}\cdot\bar{v}\cdot\bar{x}\right] = \frac{1}{\mu}\left[\hat{H}\left(x\hat{P}_x + \hat{P}_x x\right)\right] - \bar{v}^2 \tag{7}$$

sebäbi

$$\left[\hat{H}\cdot\bar{v}\cdot\bar{x}\right] = \bar{v}\left[\hat{H}\bar{x}\right] = \bar{v}\cdot\frac{d\bar{x}}{dt} = \bar{v}^2$$

(7)-niň birinji çlenini hasaplalyň:

$$\frac{1}{\mu}\left[\hat H\left(x\hat P_x+\hat P_x x\right)\right]=\frac{1}{i\hbar\mu}\left\{\left(x\hat P_x+\hat P_x x\right)\hat H-\hat H\left(x\hat P_x+\hat P_x x\right)\right\}=$$

$$=\frac{1}{2i\hbar\mu^2}\left\{\left(x\hat P_x+\hat P_x x\right)\cdot\hat P_x^2-\hat P_x^2\left(x\hat P_x+\hat P_x x\right)\right\}=\frac{1}{2i\hbar\mu^2}\left(x\hat P_x^3+\hat P_x x\hat P_x^2-\hat P_x^2 x\hat P_x-\hat P_x^3 x\right)=$$

$$=\frac{1}{2i\hbar\mu^2}\left\{x\hat P_x^3+\hat P_x x\hat P_x^2-\hat P_x\left(x\hat P_x-i\hbar\right)\hat P_x-\hat P_x^2\left(x\hat P_x-i\hbar\right)\right\}=$$

$$=\frac{1}{2i\hbar\mu}\left(x\hat P_x^3+\hat P_x x\hat P_x^2-\hat P_x x\hat P_x^2+i\hbar\hat P_x^2-\hat P_x^2 x\hat P_x+i\hbar\hat P_x^2\right)=\frac{1}{2i\hbar\mu^2}\left\{x\hat P_x^3+2i\hbar\hat P_x^2-\hat P_x\left(x\hat P_x-i\hbar\right)\cdot\hat P_x\right\}=$$

$$=\frac{1}{2i\hbar\mu}\left\{x\hat P_x^3+2i\hbar\hat P_x^2+i\hbar\hat P_x^2-\left(x\hat P_x-i\hbar\right)\cdot\hat P_x^2\right\}=\frac{1}{2i\hbar\mu^2}\left(x\hat P_x^3+3i\hbar\hat P_x^2-x\hat P_x^3+i\hbar\hat P_x^2\right)=\frac{2\hat P_x^2}{\mu^2};$$

onda (6) şeýle görnüşe eýe bolar:

$$\frac{d^2(\Delta x)^2}{dt^2}=\frac{2\hat P_x^2}{\mu^2}-\bar v^2. \qquad \textbf{(8)}$$

$\hat P_x^2$ operator $\hat H$ bilen kommutirleşýär. Şonuň üçin $(\Delta x)^2$ ululukdan alnan ähli ýokary önümler nola deň.

Indi $(\Delta x)^2$ ululugy t-niň derejesi boýunça Teýloryň hataryna dargadalyň:

$$(\Delta x)_t^2=(\Delta x)_0^2+\frac{1}{1!}\frac{d(\Delta x)^2}{dt}t+\frac{1}{2!}\frac{d^2(\Delta x)^2}{dt^2}t^2+\ldots \qquad \textbf{(9)}$$

(5) we (8) aňlatmalary (9) – a goýalyň:

$$\left(\Delta x\right)_t^2 = \left(\Delta x\right)_0^2 + \left(\frac{x\hat{P}_x + \hat{P}_x^{\,2} x}{\mu} - \overline{v}\,\overline{x}\right)\cdot t + \frac{1}{2}\left(\frac{2\hat{P}_x^{\,2}}{\mu^2} - \overline{v}^{\,2}\right)\cdot t^2 .$$

Operatordan orta baha geçeliň:

$$\overline{\left(\Delta x\right)_t^2} = \overline{\left(\Delta x\right)_0^2} + \left(\frac{\overline{xP_x + P_x x}}{\mu} - \overline{v}\,\overline{x}\right)\cdot t + \frac{1}{2}\left(\frac{\overline{2P_x^{\,2}}}{\mu^2} - \overline{v}^{\,2}\right)\cdot t^2 .$$

(10)

$\overline{\left(\Delta x\right)_t^2}$ ululyk hökmany suratda položitel ululyk. Şol sebäpli (10) – dan görnüşi ýaly, t-niň artmagy bilen $\overline{\left(\Delta x\right)_t^2}$ çäksiz artýar, ýagny tolkun paketi ýaýraýar.

Daşky güýjüň ýoklugynda, tolkun paketiniň hereketi we ýaýramagy aşakda görkezilýär.

V bap

Orta bahanyň hasaplanylyşy. Erkin bölejik üçin Şrýodingeriň deňlemesi

V.1. Usuly görkezmeler

- Kwant mehanikasynda fiziki ululugyñ orta bahasy, kesgitlemä laýyklykda şeýle tapylýar:

$$\overline{L} = \int_{-\infty}^{+\infty} \psi^* \hat{L} \psi dx$$

we Şrýodingeriň deňlemesi

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi \; .$$

bu ýerde

$$\hat{H} = \frac{\hat{P}^2}{2\mu} + \hat{U}(x, y, z, t),$$

ýa-da

$$\hat{H} = -\frac{\hbar^2}{2\mu} \nabla^2 + \hat{U}(x, y, z, t),$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

V.2. Meseleler

5.1. Tolkun funksiýasy aşakdaky görnüşde

$$\psi = e^{-i\frac{E}{\hbar}t + i\frac{\hat{P}}{\hbar}x} \qquad \textbf{(1)}$$

bolsa, Şrýodingeriň deňlemesini getirip çykarmaly.

Çözülişi:

(l) – den aşakdaky önümleri tapýarys:

$$\frac{\partial \psi}{\partial t} = -i\frac{E}{\hbar}\psi = \frac{E}{i\hbar}\psi \qquad \textbf{(2)}$$

$$\frac{\partial \psi}{\partial x} = i\frac{\hat{P}}{\hbar}\psi \qquad \text{we} \qquad \frac{\partial^2 \psi}{\partial x^2} = -\frac{\hat{P}^2}{\hbar^2}\psi \qquad \textbf{(3)}$$

(3) we (2)-ni bilelikde çözüp

$$\frac{\partial \psi}{\partial t} = -\frac{E}{i\hbar}\frac{\hbar^2}{P^2}\frac{\partial^2 \psi}{\partial x^2} = -\frac{\hbar^2}{i\hbar}\frac{E}{2\mu E}\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{i\hbar}\hat{H}\psi$$

alarys.
Şeýlelikde,

$$i\hbar\frac{\partial \psi}{\partial t} = \hat{H}\psi,$$

bu ýerde

$$\hat{H} = \frac{\hat{P}^2}{2\mu} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}$$

5.2. $t = 0$ wagt pursatynda erkin bölejigiň özüni alyp barşy

$$\psi(x,0) = Ae^{-\frac{x^2}{a^2} + ik_0 x}$$

funksiýa bilen aňladylýar. Normirleýji A koeffisiýenti, bölejigiň barlyk oblastyny we toguň j dykyzlygyny tapmaly.

Çözüwi:

Berlen ψ funksiýany

$$\int_{-\infty}^{+\infty} \left|\psi(x,0)\right|^2 dx = 1$$

şertde goýup, A -ny taparys:

$$\left|A\right|^2 \int_{-\infty}^{+\infty} e^{-2\frac{x^2}{a^2}} dx = \left|A\right|^2 \int_{-\infty}^{0} e^{-2\frac{x^2}{a^2}} dx + \left|A\right|^2 \int_{0}^{\infty} e^{-2\frac{x^2}{a^2}} dx = 2\left|A\right|^2 \int_{0}^{\infty} e^{-2\frac{x^2}{a^2}} dx = 1.$$

$$\frac{2x^2}{a^2} = y^2, \qquad x = \frac{a}{\sqrt{2}} y, \qquad dx = \frac{a}{\sqrt{2}} dy \ .$$

belgilemeleri ulanalyň.

Onda

$$2\left|A\right|^2 \int_0^{\infty} e^{-y^2} \frac{a}{\sqrt{2}} dy = 1$$

ýa-da

$$2\left|A\right|^2 \frac{a}{\sqrt{2}} \frac{\sqrt{\pi}}{2} = 1$$

sebäbi

$$\int_0^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{2}.$$

Şu ýerden

$$\left|A\right|^2 = \frac{\sqrt{2}}{a\sqrt{\pi}}.$$

Bölejigiň bar ýerini tapmak üçin, aşakdakyny tapmaly

$$\rho = \left|\psi(x,o)\right|^2 = \left|A\right|^2 e^{-\frac{2x^2}{a^2}}$$

Şu funksiýa, görnüşi ýaly, $x = 0$ nokatda iñ uly bahany alýar.

Tok dykyzlygynyň ähtimallygy

$$j_x = \frac{i\hbar}{2\mu}\left(\psi \frac{\partial \psi^*}{\partial x} - \psi^* \frac{\partial \psi}{\partial x}\right) = \left|A\right|^2 \frac{\hbar k_0}{\mu} e^{-2\frac{x^2}{a^2}} = \frac{\hbar k_0}{\mu} \rho.$$

5.3. Bölejigiň ýagdaýy

$$\psi(x) = Ae^{-\frac{x^2}{a^2}+ik_0x}$$

funksiýa bilen berlen bolsa, onuň koordinatasynyň we impulsynyň orta bahalaryny tapmaly.

Çözüwi:

$$\overline{x} = \int_{-\infty}^{+\infty} x|\psi|^2 dx = |A|^2 \int_{-\infty}^{+\infty} xe^{-\frac{2x^2}{a^2}} dx = -|A|^2 \cdot \frac{a^2}{4} \int_{-\infty}^{+\infty} e^{-\frac{2x^2}{a^2}} d\left(-\frac{2x^2}{a^2}\right) = -2|A|^2 \cdot \frac{a^2}{4} \cdot e^{-\frac{2x^2}{a^2}} \Bigg|_{-\infty}^{+\infty} = 0.$$

we

$$\overline{P} = \int_{-\infty}^{+\infty} \psi^* \hat{P}_x \psi dx = \int_{-\infty}^{+\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) Ae^{-\frac{x^2}{a^2}+ik_0x} dx = \int_{-\infty}^{+\infty} \psi^* (-i\hbar)\left(-\frac{2x}{a^2} + ik_0\right) \cdot Ae^{-\frac{x^2}{a^2}+ik_0x} dx =$$

$$= \frac{2\hbar}{a^2} \int_{-\infty}^{+\infty} \psi^* x\psi dx + \hbar k_0 \int_{-\infty}^{+\infty} \psi^* \psi dx = \hbar k_0,$$

sebäbi

$$\int_{-\infty}^{+\infty} \psi^* x\psi dx = 0$$

we

$$\int_{-\infty}^{+\infty} \psi^* \psi dx = 1.$$

5.4. Bölejigiň tolkun funksiýasy

$$\psi = A\psi^{-\frac{x^2}{a^2}+ik_0x}$$

görnüşde berlen bolsa, $\overline{\Delta x^2}$ we $\overline{\Delta P^2}$ ululyklary hasaplamaly we kesgitsizlik gatnaşygyny barlamaly.

Çözüwi:

$\Delta x = x - \overline{x}$, eger $\overline{x} = 0$ bolsa, onda $\Delta x = x$ we $(\Delta x)^2 = x^2$.

Şonuň üçin

$$\overline{(\Delta x)^2} = \int_{-\infty}^{+\infty} \psi^* x^2 \psi dx = \int_{-\infty}^{+\infty} Ae^{-\frac{x^2}{a^2}-ik_0x} \cdot x^2 Ae^{-\frac{x^2}{a^2}+ik_0x} dx = |A|^2 \int_{-\infty}^{+\infty} x^2 e^{-\frac{2x^2}{a^2}} dx = 2|A|^2 \int_0^{\infty} x^2 e^{-\frac{2x^2}{a^2}} dx \, .$$

Integraly bölekleýin çözeliň:

$$u = x, \quad du = dx \qquad \text{we} \quad dv = xe^{-\frac{2x^2}{a^2}}dx, \qquad v = -\frac{a^2}{4}e^{-\frac{2x^2}{a^2}}.$$

Onda

$$.\,\overline{(\Delta x)^2} = -\frac{2|A|^2 a^2}{4}xe^{-\frac{2x^2}{a^2}} + 2|A|^2\frac{a^2}{4}\int_0^{\infty} e^{-\frac{2x^2}{a^2}}dx = 2|A|^2\frac{a^2}{4}\frac{a}{\sqrt{2}}\frac{\sqrt{\pi}}{2} = \frac{a^2}{4}$$

$$\overline{(\Delta x)^2} = \frac{a^2}{4} \qquad \text{we} \qquad \overline{(\Delta P)^2} = \frac{\hbar^2}{a^2}.$$

Bu bahalary Geýzengeriň kesgitsizlik gatnaşygynda goýup,

$$\overline{(\Delta x)^2}\cdot\overline{(\Delta P)^2} = \frac{\hbar^2}{4}.$$

alarys.
Görnüşi ýaly agzalan gatnaşyk ýerine ýetýär.

5.5. Käbir bölejigiň ýagdaýyny aňladýan tolkun funksiýasyny

$$\psi(x,t) = \psi(x)\cdot e^{-\frac{i}{\hbar}Et}$$

görnüşde ýazyp bolar. Bölejigi tapmaklygyň ähtimallygynyň dykyzlygynyň diňe koordinat ψ – funksiýa bilen kesgitlenýändigini görkezmeli.

Berlen:	**Çözüwi:**
$\psi(x,t) = \psi(x)e^{-\frac{i}{\hbar}Et}$	$\omega = \lvert\psi(x,t)\rvert^2 = \psi^*(x,t)\cdot\psi(x,t)$
$\omega - ?$	$\omega = \psi^*(x)e^{+\frac{i}{\hbar}Et}\cdot\psi(x)e^{-\frac{i}{\hbar}Et} = \psi^*(x)\psi(x) = \lvert\psi(x)\rvert^2$

5.6. Wodorod atomynda elektronyň esasy ýagdaýy

$$\psi = A\frac{e^{-\frac{r}{a}}}{r}$$

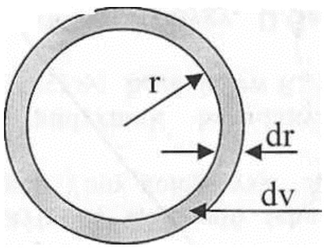
funksiýa bilen berilýär. Bu ýerde r – elektrondan ýadro çenli aralyk, a – birinji Bor radiusy. Normirleýji A koeffisiýenti kesgitlemeli.

Berlen:

$\psi(r) = A\frac{e^{-\frac{r}{a}}}{r}$,

$a = const$

$A - ?$



Çözüwi:

$$\int_v |\psi|^2 d\,v = 1,$$

$$dv = 4\pi r^2 dr,$$

$$\int_0^\infty A^2 \frac{e^{-\frac{2r}{a}}}{r^2} 4\pi^2 r^2 dr = 1,$$

$$1 = 4\pi^2 A^2 \int_0^\infty e^{-\frac{2r}{a}} dr = 4\pi^2 A^2 \left(-\frac{a}{2} e^{-\frac{2r}{a}} \right)\Bigg|_0^\infty = 4\pi^2 A^2 \frac{a}{2} = 2\pi^2 A^2 a$$

şu ýerden,
$$A = \sqrt{\frac{1}{2\pi a}}.$$

5.7. Tolkun funksiýasy $\psi = Ae^{-\frac{r}{a}}$ görnüşde berilýär. A ululygy tapmaly.

Berlen:

$\psi = Ae^{-\frac{r}{a}}$

$A - ?$

Çözüwi:

$$\int_v |\psi|^2 dv = 1, \quad dv = 4\pi r^2 dr, \quad |\psi|^2 = A^2 e^{-\frac{2r}{a}}.$$

$$1 = \int_0^\infty A^2 e^{-\frac{2r}{a}} 4\pi r^2 dr = 4\pi A^2 \int_0^\infty r^2 e^{-\frac{2r}{a}} dr = 4\pi A^2 \frac{2!}{\left(\frac{2}{a}\right)^3} =$$

$$= 4\pi A^2 \frac{a^3}{4} = \pi A^2 a^3.$$

Şu ýerden

$$A = \sqrt{\frac{1}{\pi a^3}}$$

5.8. Tolkun funksiýa $\psi = A\sin\frac{2\pi x}{l}$ diňe $0 \le x \le l$ oblastda kesgitlenipdir. Normirleýji şerti ulanyp, $"A"$ – ny kesgitlemeli.

Berlen:

$\psi = A\sin\frac{2\pi x}{l}$

$0 \le x \le l$

$A - ?$

Çözüwi:

$$\int_0^l |\psi(x)|^2 dx = 1,$$

$$|\psi(x)|^2 = A^2\sin^2\frac{2\pi x}{l}$$

$$1 = \int_0^l |\psi(x)|^2 dx = A^2\int_0^l \sin^2\frac{2\pi x}{l}dx = A^2\int_0^l \frac{1}{2}\left(1 - \cos\frac{4\pi x}{l}\right)dx =$$

$$= \frac{A^2}{2}\int_0^l dx - \frac{A^2}{2}\int_0^l \cos\frac{4\pi x}{l}dx = \frac{A^2}{2}\cdot x\Big|_0^l - \frac{A^2}{2}\cdot\frac{l}{4\pi}\sin\frac{4\pi x}{l}\Big|_0^l = \frac{A^2}{2}l.$$

şu ýerden, $A = \sqrt{\frac{2}{l}}.$

5.9. Tolkun funksiýa $\psi = A\frac{e^{-\frac{r}{a}}}{r}$. Güýç merkeze çenli aralygyň orta $\langle r \rangle$ bahasyny kesgitlemeli.

Berlen:

$\psi = A\frac{e^{-\frac{r}{a}}}{r}$

$a = const$

$< r > - ?$

Çözüwi:

$$< r >= \int_0^\infty r|\psi|^2 dv = \int_0^\infty r\psi^*\psi dv,$$

$$dv = 4\pi r^2 dr$$

$$A = \sqrt{\frac{1}{2\pi a}}$$ *(5.6-njy meselä seret)*

$$< r >= \int_0^\infty \frac{r}{2\pi a r^2}\cdot e^{-\frac{2r}{a}} 4\pi r^2 dr = \frac{2}{a}\int_0^\infty r e^{-\frac{2r}{a}} dr = \frac{2}{a}\frac{1}{\left(\frac{2}{a}\right)^2} = \frac{a}{2}.$$

Jogaby: $< r >= \frac{a}{2}$

5.10. Birölçegli tükeniksiz çuň potensial çukurda ýerleşen bölejigiň tolkun funksiýasyny we energiýasyny tapmaly.

$$-\frac{0}{2} < x > \frac{a}{2} \qquad \text{şertde } U = 0;$$

$$x > \frac{a}{2} \qquad \text{şertde } U = \infty .$$

Çözülişi:

Şrýodingeriň deňlemesi

$$\frac{d^2\psi}{dx^2} + \frac{2\mu E}{\hbar^2}\psi = 0 \qquad \textbf{(1)}$$

Meseläniň şertine görä, gyra şerti

$$\psi\left(-\frac{a}{2}\right) = \psi\left(\frac{a}{2}\right)$$

Şeýle şertde (1) – iň çözgüdi:

$$\psi = a\sin\alpha x + b\cos ax,$$

görnüşdedir.
Bu ýerde

$$a = \left(\frac{2\mu E}{\hbar^2}\right)^{\frac{1}{2}} \qquad \textbf{(2)}$$

Gyra şerti ulanyp, alarys:

$$a\sin a\left(-\frac{a}{2}\right) + b\cos a\left(-\frac{a}{2}\right) = a\sin a\left(\frac{a}{2}\right) + b\cos a\left(\frac{a}{2}\right),$$

ýa – da

$$-a\sin\frac{a\alpha}{2}(-) + b\cos\frac{a\alpha}{2} = a\sin\frac{a\alpha}{2} + b\cos\frac{a\alpha}{2},$$

Onda

$$2a\sin\frac{a\alpha}{2} = 0$$

$$\sin\frac{a\alpha}{2} = 0.$$

Diýmek ,

$$\frac{a\alpha}{2} = n\pi .$$

Şu ýerden

$$\alpha = \frac{n\pi}{a} . \qquad \textbf{(3)}$$

(2) we (3)-den:

$$E_n = \frac{\hbar^2}{2\mu}\frac{\pi^2 n^2}{a^2}$$

we

$$\psi = b\cos\frac{n\pi}{a}x$$

"b" – ni funksiýany normirleme şertinden taparys.

$$1 = \int_{-\frac{a}{2}}^{\frac{a}{2}}|\psi|^2 dx = b^2\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos^2\frac{n\pi}{a}x = b^2\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1+\cos\frac{2n\pi}{a}x}{2}dx = b^2\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{2}dx + \frac{b^2}{2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos\frac{2n\pi}{a}xdx = \frac{b^2}{2}(x)\Big|_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{b^2\cdot a}{2},$$

sebäbi

$$\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos\frac{2n\pi}{a}xdx = 0.$$

Şeýlelikde,

$$b = \sqrt{\frac{2}{a}}.$$

Gutarnykly ýagdaýda

$$\psi_n = \sqrt{\frac{2}{a}}\cos\frac{n\pi}{a}x$$

5.11. Birölçegli hereketiň impulsynyň orta bahasy aşakdaky görnüşde alnyp bilinjekdigini subut etmeli.

$$\overline{P} = \frac{\hbar}{2i}\int_{-\infty}^{+\infty}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)dx = \mu\int_{-\infty}^{+\infty}Idx .$$

Çözüwi:

Kesgitlemä görä,

$$\overline{P} = \int_{-\infty}^{+\infty} \psi^{*} \hat{P}_x \psi dx = \frac{\hbar}{i} \int_{-\infty}^{+\infty} \psi^{*} \frac{\partial}{\partial x} \psi dx$$

Ýöne, normirleme şertine görä $\int_{-\infty}^{+\infty} \psi^{*} \psi dx = 1$

$$\frac{\partial}{\partial x} \int_{-\infty}^{+\infty} \psi^{*} \psi dx = \int_{-\infty}^{+\infty} \left(\psi^{*} \frac{\partial \psi}{\partial x} + \psi \frac{\partial \psi^{*}}{\partial x} \right) dx = 0;$$

Şu ýerden

$$\int_{-\infty}^{+\infty} \psi^{*} \frac{\partial \psi}{\partial x} dx = - \int_{-\infty}^{+\infty} \psi \frac{\partial \psi^{*}}{\partial x} dx .$$

Onda:

$$\overline{P} = \frac{\hbar}{i} \cdot \frac{1}{2} \left[\int_{-\infty}^{+\infty} \psi^{*} \frac{\partial \psi}{\partial x} dx - \int_{-\infty}^{+\infty} \psi \frac{\partial \psi^{*}}{\partial x} dx \right];$$

diýip ýazyp bileris.
Ýa-da

$$\overline{P} = \frac{\hbar}{2i} \left(\int_{-\infty}^{+\infty} \psi^{*} \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^{*}}{\partial x} \right) dx$$

5.12. Normal ýagdaý *(n = 1)* üçin tolkun funksiýa aşakdaky ýaly ýazylýar:

$$\psi_1 = \sqrt{\frac{2}{a}} \cos \frac{\pi}{a} x .$$

Bölejigiň impulsynyň we impulsyň kwadratynyň orta bahalaryny tapmaly.

Çözüwi:

$$\overline{P} = \int_{-\frac{a}{2}}^{\frac{a}{2}} \psi_1^{*} \frac{\hbar}{i} \frac{d}{dx} \psi_1 dx = \frac{\hbar}{i} \frac{2\pi}{a^2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \cos \frac{\pi}{a} x \sin \frac{\pi}{a} x dx = \frac{\hbar}{i} \frac{\pi}{a^2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \sin \frac{2\pi}{a} x dx = - \frac{\hbar}{i} \frac{1}{2a} \cos \frac{2\pi}{a} x \Bigg|_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} = 0$$

we

$$\overline{P^2} = \int_{-\frac{a}{2}}^{\frac{a}{2}} \psi_1^* \frac{\hbar^2}{i^2} \frac{d^2}{dx^2} \psi_1 dx = \frac{2\pi^2 \hbar^2}{a^3} \int_{-\frac{a}{2}}^{\frac{a}{2}} \cos^2 \frac{\pi}{a} x = \frac{\hbar^2 \pi^2}{a^2}.$$

VI bap

Bölejikleriň potensial päsgelçiliklerden geçişi. Tunnel effekti

VI.1. Usuly görkezmeler

- Absolýut syzdyrmaýan birölçegli göniburçly potensial çukurdaky bölejikler üçin Şrýodingeriň stasionar ýagdaý üçin deňlemesiniň çözüwi olaryň energiýasynyň kwantlanmagyna getirýär, ýagny

$$E_n = \frac{\pi^2 \hbar^2}{2ml^2} n^2, \qquad n = 1,2,3,... \qquad \textbf{(VI.1)}$$

bu ýerde l - potensial çukuryň ini.

- diskret spektre degişli tolkun funksiýalary

$$\psi_n(x) = \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l} x \qquad \textbf{(VI.2)}$$

görnüşdedir.

- U_0 beýiklikli göniburçly potensial päsgelçilige uçýan bölejikleriň serpikme R we durulyk D koeffisiýentleri:

$$R = \left| \frac{k_1 - k_2}{k_1 + k_2} \right|^2, \qquad D = \frac{4k_1 k_2}{(k_1 + k_2)^2} \qquad \textbf{(VI.3)}$$

bu ýerde $k_1 = \sqrt{\frac{2m}{\hbar^2} E}, \qquad k_2 = \sqrt{\frac{2m}{\hbar^2} (E - U_0)}$

E - bölejikleriň energiýasy.

- Erkin $U(x)$ formaly potensial päsgelçiligiň durulyk D koeffisiýenti aşakdaky formula bilen kesgitlenilýär:

$$D = D_0 \exp\left\{ -\frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m(U(x)) - E}\, dx \right\}, \qquad \textbf{(VI.4)}$$

bu ýerde x_1 we x_2 - “giriş” we “çykyş” nokatlary, ýagny $U(x) = E$ ýagdaýdaky nokatlaryň koordinatalary.

VI.2. Meseleler

6.1. Bölejik absolýut syzdyrmaýan diwarly l - inli $(0 < x < l)$ birölçegli potensial çukuryň esasy ýagdaýynda ýerleşýär. Bölejikleriň $\left(l/3,\ 2l/3\right)$ interwalda ýerleşmekleriniň ähtimallygyny tapmaly.

Çözüwi:

Esasy ýagdaý üçin $n = 1$. Oňa degişli tolkun funksiýa (VI.2)-den alynýar:

$$\psi_1(x) = \sqrt{\frac{2}{l}} \sin \frac{\pi}{l} x$$

Berlen interwalda (x_1, x_2) bölejikleri tapmaklygyň ähtimallygy

$$\omega = \int_{x_1}^{x_2} |\psi|^2 dx = \frac{2}{l} \int_{l/3\ 22222}^{2l/3} \sin^2\left(\frac{\pi}{l} x\right) dx = \frac{1}{3} + \frac{\sqrt{3}}{2\pi} \approx 0{,}64 .$$

6.2. Absolýut syzdyrmaýan diwarly potensial çukuryň içinde "m" massaly bölejik esasy ýagdaýda ýerleşýär. Şeýle ýagdaýda bölejikleriň ýerleşmekleriniň ähtimallygynyň dykyzlygynyň maksimum bahasy ω_m. Çukuryň l inini we bölejikleriň E energiýasyny tapmaly.

Çözüwi:

Esasy ýagdaý $(n = 1)$ üçin ähtimallygyň dykyzlygynyň koordinat baglylygy

$$\omega(x) = |\psi_1|^2 = \frac{2}{l} \sin^2\left(\frac{\pi}{l} x\right)$$

ululyga deňdir.

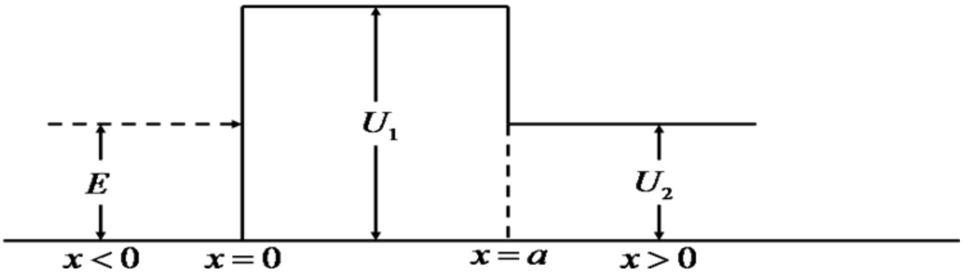
Şu taýdan görnüşi ýaly, $x = \frac{l}{2}$-de ähtimallygyň dykyzlygy maksimumdyr. Şonuň üçin

$$\omega_m = \frac{2}{l} \sin^2\left(\frac{\pi}{l} \cdot \frac{l}{2}\right) = \frac{2}{l}.$$

Şu taýdan tapylýar $l = \frac{2}{\omega_m}$ we (VI.1)-e laýyklykda

$$E_1 = \frac{\pi^2 \hbar^2}{2ml^2} \omega_m^2 .$$

6.5. Aşakdaky çyzgyda görkezilen potensial barýer üçin bölejikleriň ondan geçiş koeffisiýentini (durulyk koeffisiýentini) tapmaly.



$$U=\begin{cases}0, & x<0,\\ U_1, & 0\le x\le a,\\ U_2, & x>a.\end{cases}$$

Düşýän bölejigiň energiýasy $U_2 < E < U_1$ deňsizligi kanagatlandyrýar.

Çözüwi

Dürli oblastlar üçin Şrýodingeriň deňlemesi:

$$\frac{d^2\psi_1}{dx^2}+\frac{2\mu E}{\hbar^2}\psi_1=0, \qquad x<0,$$

$$\frac{d^2\psi_2}{dx^2}+\frac{2\mu}{\hbar^2}\left(E-U_1\right)\psi_2=0, \qquad 0\le x\le 0,$$

$$\frac{d^2\psi_3}{dx^2}+\frac{2\mu}{\hbar^2}\left(E-U_2\right)\psi_3=0, \qquad x>a.$$

Şu deňlemeleriň çözgütlerini asakdaky görnüşde ýazyp bileris.

$$\psi_1=A_1e^{ikx}+B_1e^{-ikx}, \qquad x<0,$$
$$\psi_2=A_2e^{lx}+B_2e^{-lx}, \qquad 0\le x\le 0,$$
$$\psi_3=A_3e^{\ln x}, \qquad x>0.$$

Bu ýerde

$$k=\sqrt{\frac{2\mu E}{\hbar^2}}, \qquad l=\sqrt{\frac{2\mu\left(E-U_1\right)}{\hbar^2}}, \qquad n=\sqrt{\frac{2\mu}{\hbar^2}\left(E-U_2\right)}.$$

Tolkun funksiýanyň üznüksizliginiň şertleri aşakdaky denlikleri berýär.

$$\psi_1\big|_{x=0}=\psi_2\big|_{x=0},$$

$$\left.\frac{\partial\psi_1}{\partial x}\right|_{x=0}=\left.\frac{\partial\psi_2}{\partial x}\right|_{x=0},$$

$$\psi_2\big|_{x=a} = \psi_3\big|_{x=a},$$

$$\left.\frac{\partial\psi_2}{\partial x}\right|_{x=a} = \left.\frac{\partial\psi_3}{\partial x}\right|_{x=a},$$

ýa-da

$$\left.\begin{aligned} &A_1 + B_1 = A_2 + B_2, \\ &ik\left(A_1 + B_1\right) = l\left(A_2 - B_2\right), \\ &A_2 e^{la} + B_2 e^{-la} = A_3 e^{ina}, \\ &l\left(A_2 e^{la} - B_2 e^{-la}\right) = inA_3 e^{ina}. \end{aligned}\right\}$$

Hasaplamany sadalaşdyrmak üçin mydama $ka >> 1$ deňsizlik ýerine ýetýär diýip hasap edip bolýar we e^{-la} ululyga proporsional çlenleri e^{la} ululyga proporsional çlenler bilen deňeşdireniňde iňkär edilip bilner. Onda:

$$\frac{A_3}{A_1} = \frac{4ik_1k_2}{\left(ik_1 - k_2\right)\left(k_2 - ik_3\right)} e^{-k_2 d} \cdot e^{ina}$$

alarys.

Degişlilikde geçiş koeffisiýenti

$$D = \frac{\left|A_3\right|^2}{\left|A_1\right|^2} = \frac{16\,k_1^2 k_2^2}{\left(k_1^2 + k_2^2\right)\left(k_2^2 + k_3^2\right)} e^{-2k_2 d}.$$

bolar.

6.4. Bölejik $0 \le x \le a$ birölçegli potensial çukurda ýerleşýär. Onuň içinde $U = 0$, daşynda bolsa $U = \infty$. Şeýle ýagdaý üçin Şrýodingeriň stasionar deňlemesiniň çözgüdini tapmaly.

Çözüwi:

$I\left(x < 0\right)$ we $III\left(x > 0\right)$oblastlara seredeliň. Şeýle oblastlar üçin Şrýodingeriň deňlemesi şeýle ýazylýar:

$$\frac{d^2\psi_1}{dx^2} = \frac{2\mu}{\hbar^2}\left(V - E\right)\psi_1.$$

Şu ýerden görnüşi ýaly $V = \infty$ bolanda, ψ_1 nola öwrülmeli, ýagny $\psi_1 = 0$; Edil şunuň ýaly

$$\psi_{III} = 0, \qquad \text{we} \qquad \frac{d^2\psi_{II}}{dx^2} + \frac{2\mu E}{\hbar^2}\psi_{II} = 0.$$

$\frac{2\mu E}{\hbar^2} = k^2$ belgini girizeliň , onda onuň çözgüdi:

$$A_{II} = A\sin(kx + \alpha).$$

Üznüksizligiň talaby:

$$\psi_1(0) = \psi_{II}(0),$$
$$\psi_{II}(a) = \psi_{III}(a)$$

berýär:

$$A\sin\alpha = 0,$$
$$A\sin(ka + \alpha) = 0.$$

Şeýlelikde $\alpha = 0$ we k bolsa diskret bahalary alyp bilýär:

$$k_n = \frac{n\pi}{a}, \quad \text{nirede} \quad n = 1,2...$$

Bölejigiň energiýasynyň derejeleri

$$E_n = \frac{\pi^2\hbar^2}{2\mu}n^2.$$

Normirleme şertiň esasynda alýarys:

$$1 = \int_{-\infty}^{+\infty} |\psi|^2 dx = \int_0^a |\psi_{II}|^2 dx = |A|^2 \int_0^\infty \sin^2\frac{n\pi x}{a} dx.$$

Şu ýerden tapýarys:

$$A = \sqrt{\frac{2}{a}}$$

Gutarnykly tolkun funksiýa ähli giňişlik boýunça şeýle deňlik görnüşde berilýär:

$$\psi_1 = 0, \qquad \psi_{II} = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}, \qquad \psi_{III} = 0.$$

6.5. Bölejik tükeniksiz beýik “diwarly”, $"l"$ inli, birölçegli göniburçly potensial çukurda ýerleşýär. Çukuryň çäginde $0 \le x \le l$ Şrýodengeriň deňlemesini ýazmaly we ony çözmeli:

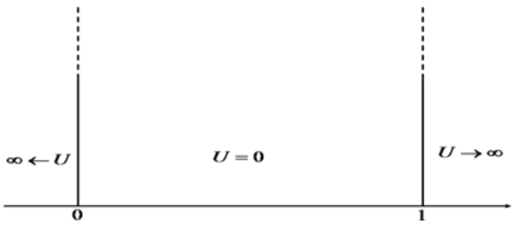
Berlen: **Çözüwi:**

$0 \le x \le l, \quad U = 0,$

$x < 0, \quad U \to \infty,$

$x > l, \quad U \to \infty.$

$\psi(x) - ?$



Differensial deňlemeleriň nazaryýetinden belli bolşy ýaly

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - U)\psi = 0, \qquad 0 \le x \le l, U = 0. \qquad \textbf{(1)}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E = 0, \qquad \textbf{(2)}$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0, \qquad \textbf{(3)}$$

$$k^2 = \frac{2m}{\hbar^2} E.$$

Bu (3) deňlemäniň çözüwi şeýle gözlenilýär:

$$\psi(x) = A \sin kx + B \cos kx, \qquad \textbf{(4)}$$

Çyzgydan $x = 0$ bolanda $\psi(0) = 0$ we $x > l$ bolanda $B = 0$

Şeýlelikde, $\psi(x) = A \sin kx$

$$\psi(l) = A \sin kl = 0, \quad kl = n\pi, \quad k = \frac{n\pi}{l}, \quad \psi_n(x) = A \sin \frac{n\pi}{l} x.$$

Jogaby: $\psi_n(x) = A \sin \dfrac{n\pi}{l} x.$

6.6. Bölejik tükeniksiz beýik „diwarly”, $"l"$ inli, birölçegli potensial çukurda ýerleşýär. Energiýanyň hususy bahasy E_n üçin aňlatmany getirip çykarmaly.

Berlen:

$\theta \le x \le l,$

$U = 0$

$E_n - ?$

Çözüwi:

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0, \quad k^2 = \frac{2mE}{\hbar^2},$$

$$k = \frac{n\pi}{l}$$ *(5.15-nji meselä seret)*

$$\frac{n^2\pi^2}{l^2} = \frac{2mE_n}{\hbar^2}, \quad E_n = n^2\frac{\pi^2\hbar^2}{2ml^2}, \quad n = 1,2,3,...$$

6.7. Tükeniksiz beýik diwarly, birölçegli göniburçly potensial çukuryň içinde elektronyň ýagdaýyny aňladýan normirlenýän hususy tolkun funksiýasynyň $\psi_n(x) = \sqrt{\frac{2}{l}}\sin\frac{n\pi}{l}x$ görnüşdedigi bellidir. Elektronyň koordinatasynyň orta bahasyny $\bar{x}$ kesgitlemeli.

Berlen:

$\psi_n(x) = \sqrt{\frac{2}{l}}\sin\frac{n\pi}{l}x$

$\bar{x} - ?$

Çözüwi:

$$\bar{x} = \int_0^l x|\psi_n(x)|^2 dx = \frac{2}{l}\int_0^l x\sin^2\frac{\pi n}{l}xdx = \frac{2}{l}\int_0^l x\left(1-\cos^2\frac{\pi n}{l}x\right)dx = \frac{1}{2}.$$

Jogaby: $\bar{x} = \frac{1}{2}.$

6.8. Bölejik tükeniksiz beýik diwarly, “l” inli, birölçegli göniburçly potensial çukurda esasy ýagdaýda ýerleşýär. Çukuryň çepinde onuň $\frac{1}{3}l$ uzaklygyndaky nokatda bölejigi tapmaklygyň ähtimallygyny kesgitlemeli.

Berlen:

$\psi_n(x) = \sqrt{\frac{2}{l}}\sin\frac{n\pi}{l}x$

$0 \le x \le \frac{l}{3}.$

$W - ?$

Çözülişi:

$$\psi_1(x) = \sqrt{\frac{2}{l}}\sin\frac{\pi}{l}x$$

$$W = \int_0^{\frac{l}{3}}|\psi_1|^2 dx = \int_0^{\frac{l}{3}}\frac{2}{l}\sin^2\frac{\pi}{l}x\cdot dx = \frac{2}{l}\int_0^{\frac{l}{3}}\frac{1}{2}\left(1-\cos\frac{2\pi}{l}x\right)dx =$$

$$= \frac{1}{l} \cdot \int_0^{\frac{l}{3}} dx - \frac{1}{l} \int_0^{\frac{l}{3}} \cos \frac{2\pi}{l} x dx = \frac{1}{3} - \frac{1}{l} \cdot \frac{l}{2\pi} \sin \frac{2\pi}{l} x \Big|_0^{\frac{l}{3}} =$$

$$= \frac{1}{3} - \frac{1}{2\pi} \sin \frac{2\pi}{3} = 0{,}195 \, .$$

Jogaby: $W = 0{,}195$.

6.9. Elektrony tükeniksiz, beýik diwarly we l inli birölçegli göniburçly potensial çukuryň ikinji $\frac{1}{3}$ böleginde tapmaklygyň ähtimallygyny tapmaly. Elektron oýandyrylan ýagdaýda (n=3) ýerleşýär diýip hasap etmeli.

Berlen:

$\psi_n(x) = \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l} x,$

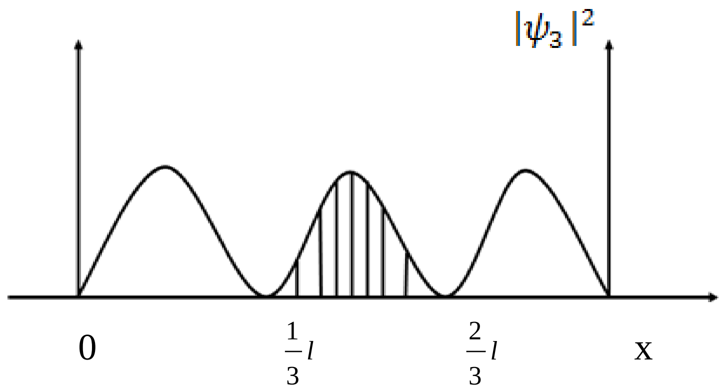
$n = 3,$

$\frac{1}{3} l \le x \le \frac{2}{3} l.$

$\omega - ?$

Çözüwi:

n



$$\psi_3(x) = \sqrt{\frac{2}{l}} \sin \frac{3\pi}{l} x$$

$$w = \int_{\frac{l}{3}}^{\frac{2l}{3}} |\psi_3|^2 dx = \int_{\frac{l}{3}}^{\frac{2l}{3}} \frac{2}{l} \sin^2 \frac{3\pi}{l} x dx = \frac{2}{l} \int_{\frac{l}{3}}^{\frac{2l}{3}} \frac{1}{2} \left(1 - \cos \frac{6\pi}{l} x \right) dx = \frac{1}{l} \int_{\frac{l}{3}}^{\frac{2l}{3}} dx - \frac{1}{l} \int_{\frac{l}{3}}^{\frac{2l}{3}} \cos \frac{6\pi}{l} x dx = \frac{1}{l} x \Big|_{\frac{l}{3}}^{\frac{2l}{3}} -$$

$$- \frac{1}{l} \frac{l}{6\pi} \sin \frac{6\pi}{l} x \Big|_{\frac{l}{3}}^{\frac{2l}{3}} = \frac{1}{3} - \frac{1}{6\pi} \left(\sin \frac{6\pi}{l} \cdot \frac{2l}{3} - \sin \frac{6\pi}{l} \cdot \frac{l}{3} \right) = \frac{1}{3} - \frac{1}{6\pi} (\sin 4\pi - \sin 2\pi) = \frac{1}{3}$$

Jogaby: $w = \frac{1}{3}$

6.10. Metalda erkin elektronyň energiýasynyň kwantlanýandygyny subut etmeli. Metalda elektron üçin potensial çukuruň inini *10sm* diýip hasap etmeli.

Berlen:

$m = 9{,}1 \cdot 10^{-31}\,kg,$

$l = 10\,sm = 0{,}1m.$

$\Delta E_n - ?$

Çözüwi:

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2ml^2} \qquad \textit{(6.8 – nji meselä seret)}$$

$$\Delta E_n = E_{n+1} - E_n = \left[(n+1)^2 - n^2\right]\frac{\pi^2\hbar^2}{2ml^2} = (2n+1)\frac{\pi^2\hbar^2}{2ml^2} \approx n\frac{\pi^2\hbar^2}{ml^2}.$$

$$\Delta E_n \approx 0{,}75\,n \cdot 10^{-16}\,eW\,.$$

Jogaby: $\Delta E_n \approx 0{,}75\,n \cdot 10^{-16}\,eW\,.$

VII bap

Çyzykly garmoniki ossilýator

VII.1.Usuly görkezmeler

- Çyzykly garmoniki ossilýatoryň kinetik we potensial energiýalarynyň operatorlary:

$$\hat{T} = \frac{1}{2m}\hat{P}^2,$$

$$\hat{U} = \frac{m_0\omega^2}{2}x^2.$$

- Umumy energiýanyň operatory:

$$\hat{H} = \frac{1}{2m}\hat{P}^2 + \frac{m_0\omega^2}{2}x^2$$

- Çyzykly garmoniki ossilýatoryň hususy energiýasy:

$$E_n = \hbar\omega_0\left(n + \frac{1}{2}\right), \qquad n = 0,1,2,...$$

- Nolunjy energiýasy:

$$E_0 = \frac{1}{2}\hbar\omega_0$$

VII.2. Meseleler

7.1. Kesgitsizlik gatnaşygyndan ugur alyp, hususy ω ýygylykly çyzykly garmoniki ossilýatoryň minimal energiýasyny bahalandyrmaly.

Çözüwi:

Çyzykly garmoniki ossilýatoryň umumy energiýasynyň nusgawy aňlatmasy.

$$E = \frac{P^2}{2m_0} + \frac{m_0\omega^2}{2}x^2 \quad \textbf{(1)}$$

Kesgitsizlik gatnaşygy

$$\Delta x \cdot \Delta P \geq \frac{\hbar}{2} \quad \textbf{(2)}$$

Hasaplamada $p \sim \Delta p$ we $x \sim \Delta x$ diýip we (2) – den Δx **-** iň bahasyny (1) – e goýup alarys:

$$E \geq \frac{(\Delta P)^2}{2m_0} + \frac{m_0\omega^2\hbar^2}{8(\Delta P)^2} \quad \textbf{(3)}$$

Şuny ekstremuma barlaýarys.

$$\frac{\partial E}{\partial (\Delta P)^2} = \frac{1}{2m_0} - \frac{m_0\omega^2\hbar^2}{8(\Delta P)^4} = 0,$$

Bu ýerden:

$$(\Delta P)^4 = \frac{m_0^2\omega^2\hbar^2}{4}$$

ýa – da

$$(\Delta P)^2 = \frac{m_0\omega\hbar}{2} \quad \textbf{(4)}$$

(4) **–** i (3) – e goýup alarys:

$$E_{\min} = \frac{1}{2}\hbar\omega$$

Bu çyzykly garmoniki ossilýatoryň kesgitsizlik gatnaşygy bilen ylalaşýan, bolup biljek iň kiçi energiýasydyr.

7.2. „n“ energetiki derejede ýerleşen çyzykly garmoniki ossilýatoryň potensial energiýasynyň orta bahasyny hasaplamaly.

Çözüwi:

$$\overline{U}_n = \frac{m_0\omega^2}{2}\overline{\left(x^2\right)_n}$$

„n“ stasionar ýagdaýy aňladýan $\psi_n(x)$ tolkun funksiýasyny hasaba alyp, ýazarys.

$$\overline{\left(x^2\right)_n} = \int_{-\infty}^{+\infty} x^2\left|\psi_n\right|^2 dx = c_n^2\left(\frac{\hbar}{m_0\omega}\right)^{\frac{3}{2}} \cdot \int_{-\infty}^{+\infty} e^{-\xi^2} H_n^2(\xi)\xi^2 d\xi \qquad \textbf{(1)}$$

bu ýerde

$$\xi = \left(\frac{m_0\omega}{\hbar}\right)^{\frac{1}{2}} \cdot x, \qquad c_n = \left(\frac{m_0\omega}{\hbar}\right)^{\frac{1}{4}} \cdot \frac{1}{\sqrt{2^n n!\sqrt{\pi}}}$$

we

$$H_n(\xi) = (-1)^n e^{\xi^2}\frac{d^n}{d\xi^n}\cdot\left(e^{-\xi^2}\right) \qquad \textbf{(2)}$$

(1) – de $H_n(\xi)$ polinomyň birini (2) – i bilen çalşyryp we n – gezek bölekleýin integrirläp alarys:

$$\overline{\left(x^2\right)_n} = c_n^2\left(\frac{\hbar}{m_0\omega}\right)^{\frac{3}{2}} \cdot \int_{-\infty}^{+\infty} e^{-\xi^2} d\xi \cdot \frac{d^n}{d\xi^n}\left(\xi^2 H_n\right) \qquad \textbf{(3)}$$

Şu formulany

$$\frac{d^n}{d\xi^n}\left(\xi^2 H_n\right) = \frac{2^n(n+2)!}{2!}\xi^2 - 2^{n-2}n(n-1)n! \qquad \textbf{(4)}$$

we

$$\int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = \sqrt{\pi}, \qquad \int_{0}^{+\infty} e^{-\xi^2} d\xi = \frac{1}{2}\sqrt{\pi}$$

integrallaryň bahalaryny hasaba alyp, taparys:

$$\overline{\left(x^2\right)_n} = \frac{\hbar}{m_0\omega}\left(n + \frac{1}{2}\right).$$

Şeýlelikde,

$$\overline{U_n} = \frac{1}{2}\hbar\omega\left(n + \frac{1}{2}\right) = \frac{1}{2}E_n.$$

7.3. Birinji oýandyrylan ýagdaýda ýerleşen çyzykly garmoniki ossilýatoryň has ähtimal ornuny tapmaly.

Çözüwi:

Ossilýatoryň tolkun funksiýasy kwant sanyň erkin bahasynyň üsti bilen aşakdaky görnüşde berilýär:

$$\psi_n = \sqrt{\frac{\alpha^{\frac{1}{2}}}{2^n n!\sqrt{\pi}}}\cdot e^{-\alpha\frac{x^2}{2}}\cdot H_n\left(\sqrt{\alpha}, x\right),$$

bu ýerde $\alpha = \dfrac{m_0\omega}{\hbar}$

we

$$H_n\left(\sqrt{\alpha}, x\right) = (-1)^n \frac{e^{\alpha x^2}}{\alpha^{\frac{n}{2}}}\cdot\frac{d^n}{dx^n}e^{-\alpha x^2}$$ – Çebyşewiň – Ermitiň polinomy.

Birinji oýandyrylan ýagdaýda *n=1* we

$$\psi_1 = \sqrt{\frac{\alpha^{\frac{1}{2}}}{2\sqrt{\pi}}}\cdot e^{-\frac{\alpha x^2}{2}}\,2\sqrt{\alpha}\cdot x = \alpha^{\frac{3}{4}}\cdot\sqrt{\frac{2}{\sqrt{\pi}}}\cdot e^{-\frac{\alpha x^2}{2}}\cdot x.$$

Ossilýatoryň ornunyň ähtimallygynyň dykyzlygy

$$|\psi|^2 = \frac{2\alpha^{\frac{3}{2}}}{\sqrt{\pi}}\cdot e^{-\alpha x^2}\cdot x^2.$$

Ähtimallygyň dykyzlygy

$$\left(e^{-\alpha x^2} \cdot x^2\right)^{/} = 0$$

şert bilen kesgitlenilýän nokatlarda maksimumy alýar. Soňky şertden

$$x = \pm\sqrt{\frac{1}{2}}$$

taparys.

7.4. Birinji oýandyrylan ýagdaýyň tolkun funksiýasy berilýär:

$$\psi_1 = \alpha^{\frac{3}{4}} \cdot \sqrt{\frac{2}{\sqrt{\pi}}} \cdot e^{-\frac{\alpha x^2}{2}} \cdot x,$$

Bu ýerde $\alpha = \frac{m\omega}{\hbar}$.

Çyzykly garmoniki ossilýatoryň doly energiasynyň orta bahasyny tapmaly.

Çözüwi:

Orta energiýa

$$\overline{H} = \int_{-\infty}^{+\infty} \psi^* \hat{H} \psi dx = \int_{-\infty}^{+\infty} \psi^* \frac{\hat{P}^2}{2m} \psi dx + \int_{-\infty}^{+\infty} \psi^* \hat{U} \psi dx = \frac{\overline{P^2}}{2m} + \overline{U} \qquad \textbf{(1)}$$

Impulsyň kwadratynyň orta bahasyny tapalyň:

$$\overline{P^2} = \int_{-\infty}^{+\infty} \psi_1^* \left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 \psi_1 dx = -\frac{\hbar^2 2\alpha^{\frac{3}{2}}}{\sqrt{\pi}} \cdot \int_{-\infty}^{+\infty} x \cdot e^{-\frac{\alpha x^2}{2}} \cdot \frac{d^2}{dx^2}\left(e^{-\frac{\alpha x^2}{2}} x\right) dx.$$

Ýöne

$$\int_{-\infty}^{+\infty} x \cdot e^{-\frac{\alpha x^2}{2}} \cdot \frac{d^2}{dx^2}\left(e^{-\frac{\alpha x^2}{2}} \cdot x\right) dx = \int_{-\infty}^{+\infty} e^{-\alpha x^2} x\left(\alpha^2 x^3 - 3\alpha x\right) dx = \frac{3\alpha^{-\frac{1}{2}}\sqrt{\pi}}{4} - \frac{3}{2}\alpha^{-\frac{1}{2}}\sqrt{\pi} = -\frac{3}{4}\frac{\sqrt{\pi}}{\alpha^{\frac{1}{2}}}.$$

Onda

$$\overline{P^2} = \frac{3\hbar^2 \alpha \cdot \sqrt{\pi}}{2\sqrt{\pi}} = \frac{3}{2}\hbar^2 \cdot \frac{m\omega}{\hbar} = \frac{3}{2}\hbar m\omega.$$

Kinetik energiýanyň orta bahasy

$$\overline{T} = \frac{\overline{P^2}}{2m} = \frac{3}{4}\hbar\omega \qquad \textbf{(2)}$$

Potensial energiýanyň orta bahasy

$$\overline{U} = \int_{-\infty}^{+\infty} \psi_1^* \hat{U} \psi_1 dx = \int_{-\infty}^{+\infty} \psi_1^* \frac{m\omega^2}{2} x^2 \psi_1 dx = \frac{m\omega^2}{2} \int_{-\infty}^{+\infty} \psi_1^* x^2 \psi_1 dx = \frac{3}{4} \frac{\sqrt{\pi}}{\left(\frac{m\omega}{\hbar}\right)^{\frac{5}{2}}} \cdot \frac{m\omega^2}{2} \frac{2}{\sqrt{\pi}} \left(\frac{m\omega}{\hbar}\right)^{\frac{3}{2}} = \frac{3}{4}\hbar\omega \qquad \textbf{(3)}$$

(2) we (3) – i (1) **–** e goýup alarys:

$$\overline{H} = \frac{3}{2}\hbar\omega$$

Bu $E_1 = \frac{3}{2}\hbar\omega$ bilen *n=1* – de gabat gelýär, sebäbi $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$.

7.5. Angarmoniki ossilýatoryň potensial energiýasy şeýle ýazylýar:

$$U(x) = \frac{m\omega_0^2}{2} x^2 + \lambda x^3 + ... \qquad \textbf{(1)}$$

(1) – iň soňky çlenleri kiçi diýip, angarmoniki ossilýatoryň kwant derejelerini tapmaly.

Çözülişi:

(1) – de tolgunma $W(x) = \lambda x^3 + ...$ **(2)**

Tolgunmadyk sistemanyň $(\lambda = 0)$ hususy energiýasy

$$E_n^0 = \hbar\omega_0\left(n + \frac{1}{2}\right) \qquad \textbf{(3)}$$

Tolgundyryjy energiýanyň matrisaly elementleri:

$$W_{mn} = \int \psi_m^{0^*} W \psi_n^0 dx = \lambda \int \psi_m^{0^*} x^3 \psi_n^0 dx = \lambda\left(x^3\right)_{mn}. \qquad \textbf{(4)}$$

Ikinji ýakynlaşdyrmada tolgundyrylan ulgamyň k derejesiniň energiýasy

$$E_k = E_k^0 + \lambda\left(x^3\right)_{kk} + \lambda^2 \sum_{n\neq k} \frac{\left(x^3\right)_{nk}\left(x^3\right)_{kn}}{E_k^0 - E_n^0} \qquad \textbf{(5)}$$

Şeýlelikde $\left(x^3\right)_{mn}$ matrisa hasaplanmaly bolýar.

Belli bolşy ýaly,

$$x_{mn} = x_0\left\{\sqrt{\frac{n}{2}}\delta_{n-1,m} + \sqrt{\frac{n+1}{2}}\delta_{n+1,m}\right\}, \qquad x_0 = \sqrt{\frac{\hbar}{m\omega_0}}. \qquad \textbf{(6)}$$

Matrisalary köpeltmek düzgüni boýunça

$$\left(x^3\right)_{kn} = \sum_l x_{kl}x_{\ln}^2 = \sum_l\sum_m x_{kl}x_{lm}x_{mn}. \qquad \textbf{(7)}$$

(7)-ä (6) **–** ny goýýarys we degişli özgertmeleri ýerine ýetirýäris:

$$\left(x^3\right)_{kn} = x_0^3\sum_l\sum_m\left\{\left(\sqrt{\frac{k}{2}}\delta_{k-1,l} + \sqrt{\frac{k+2}{2}}\delta_{k+1,l}\right)\left(\sqrt{\frac{l}{2}}\delta_{l-1,m} + \sqrt{\frac{l+1}{2}}\delta_{l+1,m}\right)\left(\sqrt{\frac{m}{2}}\delta_{m-1,n} + \sqrt{\frac{m+1}{2}}\delta_{m+1,n}\right)\right\} =$$

$$= x_0^3\sum_l\sum_m\left(\sqrt{\frac{kl}{4}}\delta_{k-1,l}\delta_{l-1,m} + \sqrt{\frac{k(l+1)}{4}}\delta_{k-1,l}\delta_{l+1,m} + \sqrt{\frac{(k+1)l}{4}}\delta_{k+1,l}\delta_{l-1,m} + \sqrt{\frac{(k+1)(l+1)}{4}}\delta_{k+1,l}\delta_{l+1,m}\right)\cdot$$

$$\cdot\left(\sqrt{\frac{m}{2}}\delta_{m-1,n} + \sqrt{\frac{m+1}{2}}\delta_{m+1,n}\right) = x_0^3\sum_m\left(\sqrt{\frac{k(k-1)}{4}}\delta_{k-2,m} + \sqrt{\frac{k^2}{4}}\delta_{km} + \sqrt{\frac{(k+1)^2}{4}}\delta_{km} + \sqrt{\frac{(k+1)(k+2)}{4}}\delta_{k+2,m}\right)\cdot$$

$$\cdot\left(\sqrt{\frac{m}{2}}\delta_{m-1,n} + \sqrt{\frac{m+1}{2}}\delta_{m+1,n}\right) = x_0^3\sum_m\left(\sqrt{\frac{k(k-1)m}{8}}\delta_{k-2,m}\delta_{m-1,n} + \sqrt{\frac{k(k-1)(m+1)}{8}}\delta_{k-2,m}\delta_{m+1,n} +\right.$$

$$+\sqrt{\frac{k^2m}{8}}\delta_{k,m}\delta_{m-1,n} + \sqrt{\frac{k^2(m+1)}{8}}\delta_{km}\delta_{m+1,n} + \sqrt{\frac{(k+1)^2m}{8}}\delta_{km}\delta_{m-1,n} + \sqrt{\frac{(k+1)^2(m+1)}{4}}\delta_{km}\delta_{m+1,n} +$$

$$\left.+\sqrt{\frac{(k+1)(k+2)m}{8}}\delta_{k+2,m}\delta_{m-1,n} + \sqrt{\frac{(k+1)(k+2)(m+1)}{8}}\delta_{k+2,m}\delta_{m+1,n}\right) = x_0^3\left(\sqrt{\frac{k(k-1)(k-2)}{8}}\delta_{k-3,n} +\right.$$

$$+\sqrt{\frac{k(k-1)^2}{8}}\delta_{k-1,n} + \sqrt{\frac{k^3}{8}}\delta_{k-1,n} + \sqrt{\frac{k^2(k+1)}{8}}\delta_{k+1,n} + \sqrt{\frac{k(k+1)^2}{8}}\delta_{k-1,n} + \sqrt{\frac{(k+1)^3}{8}}\delta_{k+1,n} +$$

$$+\sqrt{\frac{(k+1)(k+2)^2}{8}}\delta_{k+1,n}+\sqrt{\frac{(k+1)(k+2)(k+3)}{8}}\delta_{k+3,n}\Bigg)=x_3\Bigg\{\sqrt{\frac{k(k-1)(k-2)}{8}}\delta_{k-3,n}+$$

$$+\sqrt{\frac{(k+1)(k+2)(k+3)}{8}}\delta_{k+3,n}+\left(\sqrt{\frac{k(k-1)^2}{8}}+\sqrt{\frac{k^3}{8}}+\sqrt{\frac{k(k+1)^2}{8}}\right)\delta_{k-1,n}+\left(\sqrt{\frac{(k+1)k^2}{8}}+\sqrt{\frac{(k+1)^3}{8}}+\right.$$

$$\left.+\sqrt{\frac{(k+1)(k+2)^2}{8}}\right)\delta_{k+1,n}\Bigg\}=\left(\frac{\hbar}{m\omega_0}\right)^{3/2}\left(\sqrt{\frac{k(k-1)(k-2)}{8}}\delta_{k-3,n}+\sqrt{\frac{(k+1)(k+2)(k+3)}{8}}\delta_{k+3,n}+\right.$$

$$\left.+\sqrt{\frac{9k^3}{8}}\delta_{k-1,n}+\sqrt{\frac{9(k+1)^3}{8}}\delta_{k+1,n}\right). \qquad \textbf{(8)}$$

Şu taýdan görnüşi ýaly $(x^3)_{kk}=0.$.

(8) – den diňe dört çlen galýar:

$n=k\pm 3$ we $n=k\pm 1$, üstesine-de $(x^3)_{kn}=(x^3)_{nk}$.

Mundan beýläk aşakdaky aňlatmany-da hasaba almaly:

$$E_k^0-E_n^0=\hbar\omega_0\left(k+\frac{1}{2}-n-\frac{1}{2}\right)=\hbar\omega_0(k-n). \qquad \textbf{(9)}$$

Şu ýerden $n=k\pm 1$ bolanda $E_k^0-E_{k\pm 3}=\pm 3\hbar\omega_0$ we $n=k\pm 1$ bolanda

$$E_k^0-E_{k\pm 1}=\pm\hbar\omega_0.$$

Onda (8) – i (5) – e goýup alarys:

$$E_k=\hbar\omega_0\left(k+\frac{1}{2}\right)-\frac{\lambda^2}{\hbar\omega_0}\left(\frac{\hbar}{m\omega_0}\right)^3\left\{\frac{1}{3}\cdot\frac{(k+1)(k+2)(k+3)}{8}-\frac{1}{3}\cdot\frac{k(k-1)(k-2)}{8}+\frac{9}{8}(k+1)^3-\frac{9}{8}k^3\right\}=$$

$$E_k=\hbar\omega_0\left(k+\frac{1}{2}\right)-\frac{\lambda^2}{\hbar\omega_0}\left(\frac{\hbar}{m\omega_0}\right)^3\left\{\frac{1}{3}\cdot\frac{(k+1)(k+2)(k+3)}{8}-\frac{1}{3}\cdot\frac{k(k-1)(k-2)}{8}+\frac{9}{8}(k+1)^3-\frac{9}{8}k^3\right\}=$$

$$=\hbar\omega_0\left(k+\frac{1}{2}\right)-\frac{\lambda^2}{\hbar\omega_0}\left(\frac{\hbar}{m\omega_0}\right)^3\cdot\left\{\frac{1}{3\cdot 8}\left(k^3+6k^2+11k+6-k^3+3k^2-2k\right)+\frac{9}{8}\left(k^3+3k^2+3k+1-k^3\right)\right\}=$$

$$=\hbar\omega_0\left(k+\frac{1}{2}\right)-\frac{\lambda^2}{\hbar\omega_0}\left(\frac{\hbar}{m\omega_0}\right)^3\left\{\frac{1}{3\cdot 8}\left[\left(9k^2+9k+6\right)+\frac{9}{8}\left(3k^2+3k+1\right)\right]\right\}=\hbar\omega_0\left(k+\frac{1}{2}\right)-\frac{\lambda^2}{\hbar\omega_0}\left(\frac{\hbar}{m\omega_0}\right)^3\cdot$$

$$\cdot\left\{\frac{1}{3\cdot 8}\left[9k^2+9k+6+27\left(3k^2+3k+1\right)\right]\right\}=\hbar\omega_0\left(k+\frac{1}{2}\right)-\frac{\lambda^2}{\hbar\omega_0}\left(\frac{\hbar}{m\omega_0}\right)^3\left\{\frac{1}{8}\left(3k^2+3k+2+27k^2+27k+9\right)\right\}.$$

Ahyrky netije

$$E_k=\hbar\omega_0\left(k+\frac{1}{2}\right)-\frac{\lambda^2}{\hbar\omega_0}\left(\frac{\lambda}{m\omega_0}\right)^3\cdot\frac{15}{4}\left(k^2+k+\frac{11}{30}\right),$$

bu ýerde *k=0,1,2,..*

VIII bap

Puassonyň kwant skobkalary

VIII.1. Usuly görkezmeler

- Umumy energiýanyň $\hat{H}$ operatory we islendik L fiziki ululygyň $\hat{L}$ operatory üçin Gamiltonyň kwant skobkasy şeýle ýazylýar:

$$\left[\hat{H}\hat{L}\right] = \frac{1}{i\hbar}\left(\hat{L}\hat{H} - \hat{H}\hat{L}\right).$$

bu ýerde $\hat{L}$ -iň deregine islendik ululyklar bolup biler.

- Islendik $F = F(x, y, z)$ funksiýa bilen impulsyň operatorynyň $\hat{P}\left(\hat{P}_x, \hat{P}_y, \hat{P}_z\right)$ arasyndaky kommutatorlar

$$F\hat{P}_x - \hat{P}_x F = i\hbar\frac{\partial F}{\partial x},$$
$$F\hat{P}_y - \hat{P}_y F = i\hbar\frac{\partial F}{\partial y},$$
$$F\hat{P}_z - \hat{P}_z F = i\hbar\frac{\partial F}{\partial z}$$

deňdirler.

VIII.2. Meseleler

8.1. Puassonyň aşakdaky kwant skobkalaryny subut etmeli.

a. $\left[\hat{P}_y^2 \cdot y\right] = 2\hat{P}_y$.

b. $\left[\hat{P}_y \cdot y^2\right] = 2y$.

ç. $\left[\hat{A}\hat{P} \cdot x\right] = \hat{A}_x$.

d. $\left[\hat{A}\hat{P} \cdot \hat{P}_x\right] = -\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x + \frac{\partial \hat{A}_y}{\partial y}\hat{P}_y + \frac{\partial \hat{A}_z}{\partial z}\hat{P}_z\right).$

e. $\left[div\,\hat{A} \cdot \hat{P}_x\right] = -\left(\frac{\partial^2 \hat{A}_x}{\partial x^2} + \frac{\partial^2 \hat{A}_y}{\partial x \partial y} + \frac{\partial^2 \hat{A}_z}{\partial x \partial z}\right).$

ä. $\left[(ev + U)\cdot \hat{P}_x\right] = -e\frac{\partial v}{\partial x} - \frac{\partial U}{\partial x}..$

f. $\left[\hat{A}\hat{P} \cdot \hat{A}_x\right] = \hat{A}_x\frac{\partial \hat{A}_x}{\partial x} + \hat{A}_y\frac{\partial \hat{A}_x}{\partial y} + \hat{A}_z\frac{\partial \hat{A}_x}{\partial z}.$

g. $\left[\hat{P}^2\hat{A}_x\right] = -2\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x + \frac{\partial \hat{A}_y}{\partial y}\hat{P}_y + \frac{\partial \hat{A}_z}{\partial z}\hat{P}_z\right) - i\hbar\nabla^2\hat{A}_x.$

h. $\left[\hat{A}^2 \cdot \hat{P}_x\right] = -2\left(\hat{A}_x\frac{\partial \hat{A}_x}{\partial x} + \hat{A}_y\frac{\partial \hat{A}_y}{\partial x} + \hat{A}_z\frac{\partial \hat{A}_z}{\partial x}\right).$

Çözüwi:

d. $\left[\hat{A}\hat{P} \cdot \hat{P}_x\right] = \frac{1}{i\hbar}\left(\hat{P}_x \cdot \hat{A}\hat{P} - \hat{A}\hat{P} \cdot \hat{P}_x\right) = \frac{1}{i\hbar}\left(\hat{P}_x\hat{A}\hat{P} - \hat{A}_x\hat{P}_x\hat{P}_x - \hat{A}_y\hat{P}_y\hat{P}_x - \hat{A}_z\hat{P}_z\hat{P}_x\right) =$

$$= \frac{1}{i\hbar}\left\{\hat{P}_x\hat{A}\hat{P} - \left(\hat{P}_x\hat{A}_x + i\hbar\frac{\partial \hat{A}_x}{\partial x}\right)\hat{P}_x - \left(\hat{P}_x\hat{A}_y + i\hbar\frac{\partial \hat{A}_y}{\partial x}\right)\hat{P}_y - \left(\hat{P}_x\hat{A}_z + i\hbar\frac{\partial \hat{A}_z}{\partial x}\right)\hat{P}_z\right\} =$$

$$= \frac{1}{i\hbar}\left(\hat{P}_x\hat{A}_x\hat{P}_x + \hat{P}_x\hat{A}_y\hat{P}_y + \hat{P}_x\hat{A}_z\hat{P}_z - \hat{P}_x\hat{A}_x\hat{P}_x - i\hbar\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x - \hat{P}_x\hat{A}_y\hat{P}_y - i\hbar\frac{\partial \hat{A}_y}{\partial x}\hat{P}_y - \hat{P}_x\hat{A}_z\hat{P}_z - i\hbar\frac{\partial \hat{A}_z}{\partial x}\hat{P}_z\right) =$$

$$= -\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x + \frac{\partial \hat{A}_y}{\partial x}\hat{P}_y + \frac{\partial \hat{A}_z}{\partial x}\hat{P}_z\right).$$

Şeýlelikde,

$$\left[\hat{A}\hat{P}\cdot\hat{P}_x\right] = -\left(\frac{\partial\hat{A}_x}{\partial x}\hat{P}_x + \frac{\partial\hat{A}_y}{\partial x}\hat{P}_y + \frac{\partial\hat{A}_z}{\partial x}\hat{P}_z\right).$$

f. $\left[\hat{A}\hat{P}\cdot\hat{A}_x\right] = \frac{1}{i\hbar}\left(\hat{A}_x\cdot\hat{A}\hat{P} - \hat{A}\hat{P}\cdot\hat{A}_x\right) = \frac{1}{i\hbar}\left(\hat{A}_x\cdot\hat{A}\hat{P} - \hat{A}_x\hat{P}_x\hat{A}_x - \hat{A}_y\hat{P}_y\hat{A}_x - \hat{A}_z\hat{P}_z\hat{A}_x\right) =$

$$= \frac{1}{i\hbar}\left\{\hat{A}_x\hat{A}\hat{P} - \hat{A}_x\left(\hat{A}_x\hat{P}_x - i\hbar\frac{\partial\hat{A}_x}{\partial x}\right) - \hat{A}_y\left(\hat{A}_x\hat{P}_y - i\hbar\frac{\partial\hat{A}_x}{\partial y}\right) - \hat{A}_z\left(\hat{A}_x\hat{P}_z - i\hbar\frac{\partial\hat{A}_x}{\partial z}\right)\right\} =$$

$$= \frac{1}{i\hbar}\left(\hat{A}_x\hat{A}_x\hat{P}_x + \hat{A}_x\hat{A}_y\hat{P}_y + \hat{A}_x\hat{A}_z\hat{P}_z - \hat{A}_x\hat{A}_x\hat{P}_x + i\hbar\hat{A}_x\frac{\partial\hat{A}_x}{\partial x} - \hat{A}_y\hat{A}_x\hat{P}_y + i\hbar\hat{A}_y\frac{\partial\hat{A}_x}{\partial y} - \hat{A}_z\hat{A}_x\hat{P}_z + i\hbar\hat{A}_z\frac{\partial\hat{A}_x}{\partial z}\right) =$$

$$= \hat{A}_x\frac{\partial\hat{A}_x}{\partial x} + \hat{A}_y\frac{\partial\hat{A}_x}{\partial y} + A_z\frac{\partial\hat{A}_x}{\partial z}.$$

Diýmek,

$$\left[\hat{A}\hat{P}\cdot\hat{A}_x\right] = \hat{A}_x\frac{\partial\hat{A}_x}{\partial x} + \hat{A}_y\frac{\partial\hat{A}_x}{\partial y} + \hat{A}_z\frac{\partial\hat{A}_x}{\partial z}.$$

g. $\left[\hat{P}^2\hat{A}_x\right] = \frac{1}{i\hbar}\left(\hat{A}_x\hat{P}^2 - \hat{P}^2\hat{A}_x\right) = \frac{1}{i\hbar}\left(\hat{A}_x\hat{P}^2 - \hat{P}_x^2\hat{A}_x - \hat{P}_y^2\hat{A}_x - \hat{P}_z^2\hat{A}_x\right) =$

$$= \frac{1}{i\hbar}\left\{\hat{A}_x\hat{P}^2 - \hat{P}_x\left(\hat{A}_x\hat{P}_x - i\hbar\frac{\partial\hat{A}_x}{\partial x}\right) - \hat{P}_y\left(\hat{A}_x\hat{P}_y - i\hbar\frac{\partial\hat{A}_x}{\partial y}\right) - \hat{P}_z\left(\hat{A}_x\hat{P}_z - i\hbar\frac{\partial\hat{A}_x}{\partial z}\right)\right\} =$$

$$= \frac{1}{i\hbar}\left(\hat{A}_x\hat{P}^2 - \hat{P}_x\hat{A}_x\hat{P}_x + i\hbar\hat{P}_x\frac{\partial\hat{A}_x}{\partial x} - \hat{P}_y\hat{A}_x\hat{P}_y + i\hbar\hat{P}_y\frac{\partial\hat{A}_x}{\partial y} - \hat{P}_z\hat{A}_x\hat{P}_z + i\hbar\hat{P}_z\frac{\partial\hat{A}_x}{\partial z}\right) =$$

$$= \frac{1}{i\hbar}\left\{\hat{A}_x\hat{P}^2 - \left(\hat{A}_x\hat{P}_x - i\hbar\frac{\partial\hat{A}_x}{\partial x}\right)\hat{P}_x + i\hbar\hat{P}_x\frac{\partial\hat{A}_x}{\partial x} - \left(\hat{A}_x\hat{P}_y - i\hbar\frac{\partial\hat{A}_x}{\partial y}\right)\hat{P}_y + i\hbar\hat{P}_y\frac{\partial\hat{A}_x}{\partial y} - \left(\hat{A}_x\hat{P}_z - i\hbar\frac{\partial\hat{A}_x}{\partial z}\right)\hat{P}_z +\right.$$

$$\left. + i\hbar\hat{P}_z\frac{\partial\hat{A}_x}{\partial z}\right\} = \frac{1}{i\hbar}\left(\hat{A}_x\hat{P}_x^2 + \hat{A}_x\hat{P}_y^2 + \hat{A}_x\hat{P}_z^2 - \hat{A}_x\hat{P}_x\hat{P}_x + i\hbar\frac{\partial\hat{A}_x}{\partial x}\hat{P}_x + i\hbar\hat{P}_x\frac{\partial\hat{A}_x}{\partial x} - \hat{A}_x\hat{P}_y\hat{P}_y + i\hbar\frac{\partial\hat{A}_x}{\partial y}\hat{P}_y +\right.$$

$$\left. + i\hbar\hat{P}_y\frac{\partial\hat{A}_x}{\partial y} - \hat{A}_x\hat{P}_z\hat{P}_z + i\hbar\frac{\partial\hat{A}_x}{\partial z}\hat{P}_z + i\hbar\hat{P}_z\frac{\partial\hat{A}_x}{\partial z}\right) = \frac{\partial\hat{A}_x}{\partial x}\hat{P}_x + \frac{\partial\hat{A}_x}{\partial x}\hat{P}_x - \frac{\partial\hat{A}_x}{\partial x}\hat{P}_x + \hat{P}_x\frac{\partial\hat{A}_x}{\partial x} + \frac{\partial\hat{A}_x}{\partial y}\hat{P}_y +$$

$$+\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y-\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y+\hat{P}_y\frac{\partial \hat{A}_x}{\partial y}+\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z+\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z-\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z+\hat{P}_z\frac{\partial \hat{A}_x}{\partial z}=2\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x+\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y+\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z\right)+$$

$$+\left(\hat{P}_x\frac{\partial \hat{A}_x}{\partial x}-\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x\right)+\left(\hat{P}_y\frac{\partial \hat{A}_x}{\partial y}-\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y\right)+\left(\hat{P}_z\frac{\partial \hat{A}_x}{\partial z}-\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z\right)=2\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x+\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y+\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z\right)-$$

$$-i\hbar\frac{\partial}{\partial x}\cdot\frac{\partial \hat{A}_x}{\partial x}-i\hbar\frac{\partial}{\partial y}\cdot\frac{\partial \hat{A}_x}{\partial y}-i\hbar\frac{\partial}{\partial z}\cdot\frac{\partial \hat{A}_x}{\partial z}=2\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x+\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y+\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z\right)-i\hbar\frac{\partial^2 \hat{A}_x}{\partial x^2}-i\hbar\frac{\partial^2 \hat{A}_x}{\partial y^2}-i\hbar\frac{\partial^2 \hat{A}_x}{\partial z^2}=$$

$$=2\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x+\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y+\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z\right)-i\hbar\nabla^2\hat{A}_x.$$

Şeýlelikde,

$$\left[\hat{P}^2\hat{A}_x\right]=2\left(\frac{\partial \hat{A}_x}{\partial x}\hat{P}_x+\frac{\partial \hat{A}_x}{\partial y}\hat{P}_y+\frac{\partial \hat{A}_x}{\partial z}\hat{P}_z\right)-i\hbar\nabla^2\hat{A}_x$$

h. $$\left[\hat{A}^2\hat{P}_x\right]=\frac{1}{i\hbar}\left(\hat{P}_x\hat{A}^2-\hat{A}^2\hat{P}_x\right)=\frac{1}{i\hbar}\left(\hat{P}_x\hat{A}^2-\hat{A}_x^2\hat{P}_x-\hat{A}_y^2\hat{P}_x-\hat{A}_z^2\hat{P}_x\right)=\frac{1}{i\hbar}\left\{\hat{P}_x\hat{A}^2-\hat{A}_x\left(\hat{P}_x\hat{A}_x+i\hbar\frac{\partial \hat{A}_x}{\partial x}\right)-\right.$$

$$\left.-\hat{A}_y\left(\hat{P}_x\hat{A}_y+i\hbar\frac{\partial \hat{A}_y}{\partial x}\right)-\hat{A}_z\left(\hat{P}_x\hat{A}_z+i\hbar\frac{\partial \hat{A}_z}{\partial x}\right)\right\}=\frac{1}{i\hbar}\left(\hat{P}_x\hat{A}^2-\hat{A}_x\hat{P}_x\hat{A}_x-i\hbar\hat{A}_x\frac{\partial \hat{A}_x}{\partial x}-\hat{A}_y\hat{P}_x\hat{A}_y-i\hbar\hat{A}_y\frac{\partial \hat{A}_y}{\partial x}-\right.$$

$$\left.-\hat{A}_z\hat{P}_x\hat{A}_z-i\hbar\hat{A}_z\frac{\partial \hat{A}_z}{\partial x}\right)=\frac{1}{i\hbar}\left\{\hat{P}_x\hat{A}^2-\left(\hat{P}_x\hat{A}_x+i\hbar\frac{\partial \hat{A}_x}{\partial x}\right)\hat{A}_x-i\hbar\hat{A}_x\frac{\partial \hat{A}_x}{\partial x}-\left(\hat{P}_x\hat{A}_y+i\hbar\frac{\partial \hat{A}_y}{\partial x}\right)\hat{A}_y-i\hbar\hat{A}_y\frac{\partial \hat{A}_y}{\partial x}-\right.$$

$$\left.-\left(\hat{P}_x\hat{A}_z+i\hbar\frac{\partial \hat{A}_z}{\partial x}\right)\hat{A}_z-i\hbar A_z\frac{\partial \hat{A}_z}{\partial x}\right\}=\frac{1}{i\hbar}\left(\hat{P}_x\hat{A}_x^2+\hat{P}_x\hat{A}_y^2+\hat{P}_x\hat{A}_z^2-\hat{P}_x\hat{A}_x^2-i\hbar\frac{\partial \hat{A}_x}{\partial x}\cdot\hat{A}_x-i\hbar\hat{A}_x\frac{\partial \hat{A}_x}{\partial x}-\hat{P}_x\hat{A}_y^2-\right.$$

$$\left.-i\hbar\frac{\partial \hat{A}_y}{\partial x}\hat{A}_y-i\hbar\hat{A}_y\frac{\partial \hat{A}_y}{\partial x}-\hat{P}_z\hat{A}_z^2-i\hbar\frac{\partial \hat{A}_z}{\partial x}\hat{A}_z-i\hbar\hat{A}_z\frac{\partial \hat{A}_z}{\partial x}\right)=-2\left(\hat{A}_x\frac{\partial \hat{A}_x}{\partial x}+\hat{A}_y\frac{\partial \hat{A}_y}{\partial x}+\hat{A}_z\frac{\partial \hat{A}_z}{\partial x}\right).$$

Şeýlelikde,

$$\left[\hat{A}^2\cdot\hat{P}_x\right]=-2\left(\hat{A}_x\frac{\partial \hat{A}_x}{\partial x}+\hat{A}_y\frac{\partial \hat{A}_y}{\partial x}+\hat{A}_z\frac{\partial \hat{A}_z}{\partial x}\right).$$

IX bap

Elektronyň spininiň operatory. Pauliniň matrisalary we olaryň algebrasy

IX.1. Usuly görkezmeler

- Kwant mehanikasynyň prinsipine laýyklykda, s ululyga çyzykly özüneçatrymly $\hat{s}$ operatory degişlidir. Şu operatoryň $\hat{s}_x, \hat{s}_y, \hat{s}_z$ düzüjileri

$$\hat{S}_x = \frac{\hbar}{2}\hat{\sigma}_x, \qquad \hat{S}_y = \frac{\hbar}{2}\hat{\sigma}_y, \qquad \hat{S}_z = \frac{\hbar}{2}\hat{\sigma}_z.$$

bu ýerde

$$\hat{\sigma}_x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$ - Pauliniň matrisalary.

$\hat{S}_x$, $\hat{S}_y$ we $\hat{s}_z$ proýeksiýalar $\hat{M}_x$, $\hat{M}_y$ we $\hat{M}_z$ proýeksiýalaryň kanagatlandyrýan kommutasiýa düzgünine boýun egýärler:

$$\left.\begin{aligned} \hat{S}_x\hat{S}_y - \hat{S}_y\hat{S}_x &= i\hbar\hat{S}_z, \\ \hat{S}_y\hat{S}_z - \hat{S}_z\hat{S}_y &= i\hbar\hat{S}_x, \\ \hat{S}_z\hat{S}_x - \hat{S}_x\hat{S}_z &= i\hbar\hat{S}_y \end{aligned}\right\}$$

- Impuls we spin momentleriň jemi, ýagny umumy moment deňdir:

$$\hat{I} = \hat{M} + \hat{S}.$$

Onuň düzüjileri

$$\hat{I}_x = \hat{M}_x + \hat{S}_x, \quad \hat{I}_y = \hat{M}_y + \hat{S}_y, \quad \hat{I}_z = \hat{M}_z + \hat{S}_z.$$

- Umumy momentiň kwadraty

$$\hat{I}^2 = \left(\hat{M} + \hat{S}\right)^2 = \hat{M}^2 + \hat{S}^2 + 2\left(\hat{M}\hat{S}\right) = \hat{M}^2 + \hat{S}^2 + 2\left(\hat{M}_x\hat{S}_x + \hat{M}_y\hat{S}_y + \hat{M}_z\hat{S}_z\right).$$

IX.2. Meseleler

9.1. Pauliniň matrisalaryny ulanyp, spiniň operatorlarynyň düzüjileri üçin çalyşma gatnaşyklary almaly.

Çözüwi:

$\hat{M}_x\hat{M}_y - \hat{M}_y\hat{M}_x = -i\hbar\hat{M}_z$ we başga gatnaşyklarda $\hat{M}$ -i $\hat{S}$ operatora çalşyryp alarys:

$$\left.\begin{aligned} \hat{S}_x\hat{S}_y - \hat{S}_y\hat{S}_x &= i\hbar\hat{S}_z, \\ \hat{S}_y\hat{S}_z - \hat{S}_z\hat{S}_y &= i\hbar\hat{S}_x, \\ \hat{S}_z\hat{S}_x - \hat{S}_x\hat{S}_z &= i\hbar\hat{S}_y. \end{aligned}\right\} \qquad \textbf{(1)}$$

bu ýerde

$$\hat{S}_x = \frac{\hbar}{2}\hat{\sigma}_x, \quad \hat{S}_y = \frac{\hbar}{2}\hat{\sigma}_y, \quad \hat{S}_z = \frac{\hbar}{2}\hat{\sigma}_z. \qquad \textbf{(2)}$$

we

$$\hat{\sigma}_x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$ - Pauliniň matrisalary.

Şu ýerden $\hat{\sigma}_x^2 = \hat{\sigma}_y^2 = \hat{\sigma}_z^2 = 1$ gelip çykýar.

(2) – ni (1) – e goýup we $\frac{\hbar^2}{4}$ ululyga gysgaldyp alarys.

$$\left.\begin{aligned} \hat{\sigma}_x\hat{\sigma}_y - \hat{\sigma}_y\hat{\sigma}_z &= 2i\hat{\sigma}_z, \\ \hat{\sigma}_y\hat{\sigma}_z - \hat{\sigma}_z\hat{\sigma}_y &= 2i\hat{\sigma}_x, \\ \hat{\sigma}_z\hat{\sigma}_x - \hat{\sigma}_x\hat{\sigma}_z &= -2i\hat{\sigma}_y. \end{aligned}\right\}$$

9.2. Pauliniň matrisalarynyň bir-biri bilen antikommutirleşýändigini we her matrisanyň kwadratynyň bire deňdigini görkezmeli.

Çözüwi:

$$\hat{\sigma}_x\hat{\sigma}_y + \hat{\sigma}_y\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = 0,$$

diýmek, $\hat{\sigma}_x\hat{\sigma}_y = -\hat{\sigma}_y\hat{\sigma}_x$.

$$\hat{\sigma}_x^2 = \hat{\sigma}_x \cdot \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1.$$

9.3. Islendik ugra elektron spininiň düzüjisiniň kwadratyny hasaplamaly.

Çözüwi:

Belli bolşy ýaly,

$$\hat{S} = \frac{\hbar}{2}\hat{\sigma}$$

bu ýerde $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ - Pauliniň matrisalary bolup, aşakdaky şertleri kanagatlandyrýarlar:

$$\hat{\sigma}_x^2 = \hat{\sigma}_y^2 = \hat{\sigma}_z^2 = 1;,$$

$$\hat{\sigma}_x\hat{\sigma}_y = -\hat{\sigma}_y\sigma_x; \quad \hat{\sigma}_y\hat{\sigma}_z = -\hat{\sigma}_z\sigma_y; \quad \hat{\sigma}_z\hat{\sigma}_x = -\hat{\sigma}_x\hat{\sigma}_z.$$

$\vec{a}$ ugra $\hat{\vec{s}}$ -ň düzüjisi $\left(\frac{\vec{S} \cdot \vec{a}}{a}\right)$.

Şu ululygy kwadrata götereliň:

$$\left(\frac{\hat{\vec{S}}\vec{a}}{a}\right)^2 = \frac{\hbar^2}{4a^2}(\vec{\sigma}\vec{a})^2 = \frac{\hbar^2}{4a^2}\left(\hat{\sigma}_x a_x + \hat{\sigma}_y a_y + \hat{\sigma}_z a_z\right)\left(\hat{\sigma}_x a_x + \hat{\sigma}_y a_y + \hat{\sigma}_z a_z\right) =$$

$$= \frac{\hbar^2}{4a^2}\left[\hat{\sigma}_x^2\hat{a}_x^2 + \hat{\sigma}_y^2\hat{a}_y^2 + \hat{\sigma}_z^2\hat{a}_z^2 + \left(\hat{\sigma}_x\hat{\sigma}_y + \hat{\sigma}_y\hat{\sigma}_x\right)a_x a_y + \left(\hat{\sigma}_y\hat{\sigma}_z + \hat{\sigma}_z\hat{\sigma}_y\right)a_y a_z + \left(\hat{\sigma}_z\hat{\sigma}_x + \hat{\sigma}_x\hat{\sigma}_z\right)a_z a_x\right] = \frac{3}{4}\hbar^2.$$

9.4. Triplet we singlet ýagdaýlarda iki bölegiň spinleriniň skalýar köpeltmek hasylyny tapmaly.

Çözüwi:

Spiniň operatoryny girizeliň.

$$\hat{\vec{S}}_1 = \frac{\hbar}{2}\sigma_1 \quad \text{we} \quad \hat{\vec{S}}_2 = \frac{\hbar}{2}\sigma_2,$$

üstesine – de $\hat{\sigma}_i^2 = 3$, $\hat{\sigma}_{ix}^2 = \hat{\sigma}_{iy}^2 = \hat{\sigma}_{iz}^2 = 1$ aňlatmalary hasaba alyp, operatorlaryň jeminiň kwadratyna seredeliň:

$$\hat{\vec{S}}^2 = \left(\hat{S}_1 + \hat{S}_2\right)^2 = \hat{S}_1^2 + \hat{S}_2^2 + 2\left(\hat{S}_1\hat{S}_2\right). \qquad \textbf{(1)}$$

$\hat{\hat{S}}^2$, operatorynyň hususy bahalary

$$\hat{\hat{S}}^2 = \hbar^2 S(S+1),$$

$S = 1$ triplet we $S = 0$ singlet ýagdaýlar üçin,

$$\vec{S}_1^2 = \vec{S}_2^2 = \frac{3}{4}\hbar^2.$$

(1) – den:

$$\left(\hat{S}_1\hat{S}_2\right) = \frac{\hat{S}^2 - \left(\hat{S}_1^2 + \hat{S}_2^2\right)}{2} = \frac{\hbar^2}{4}\left[2S(S+1) - 3\right],$$

ýa – da

$$\left(\hat{S}_1\hat{S}_2\right) = \begin{cases} \dfrac{\hbar^2}{4}, & S = 1 \\ -\dfrac{3\hbar^2}{4}, & S = 0 \end{cases}$$

9.5. $\hat{\sigma}_x\alpha = \beta$; $\hat{\sigma}_y\alpha = i\beta$; $\hat{\sigma}_z\alpha = \alpha$; $\hat{\sigma}_x\beta = \alpha$; $\hat{\sigma}_y\beta = -i\alpha$; $\hat{\sigma}_z\beta = -\beta$; bu ýerde α we β - spin tolkun funksiýalary, deňlikler bilen kesgitlenilýän $\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$ operatorlary Pauliniň matrisalarynyň kanagatlandyrýan gatnaşyklaryna boýun egýändiklerini subut etmeli.

Çözüwi:

$\hat{\sigma}_z\alpha = \alpha$ we $\hat{\sigma}_z\beta = -\beta$ deňliklerden $\hat{\sigma}_z = \pm 1$. $\hat{\sigma}_z$ üçin hususy funksiýalary $(\alpha$ we $\beta)$ $\hat{\sigma}_x$ we $\hat{\sigma}_y$ üçin hususy funksiýalar däldirler, ýöne

$$\begin{matrix} \hat{\sigma}_x\alpha = \beta; & \hat{\sigma}_y\alpha = i\beta; & & \hat{\sigma}_y\alpha = i\beta; \\ \ddot{\sigma}_x\beta = \alpha; & \hat{\sigma}_y\beta = -i\alpha & \text{ýa-da} & \hat{\sigma}_y i\beta = \alpha, \end{matrix}$$

deňlikleri goşup we aýryp, tapýarys

$$\hat{\sigma}_x(\alpha+\beta)=\alpha+\beta; \qquad \hat{\sigma}_y(\alpha+i\beta)=\alpha+i\beta;$$
$$\hat{\sigma}_x(\alpha-\beta)=-(\alpha-\beta); \qquad \hat{\sigma}_y(\alpha-i\beta)=-(\alpha-i\beta).$$

Bulardan görnüşi ýaly, $\hat{\sigma}_x=\pm1$ we $\hat{\sigma}_y=\pm1.$

Aşakdaky aňlatmalary düzeliň:

$$(\hat{\sigma}_x\hat{\sigma}_z-\hat{\sigma}_z\hat{\sigma}_x)\alpha=\hat{\sigma}_x\alpha-\hat{\sigma}_z\beta=\beta-(-\beta)=2\beta=2i\hat{\sigma}_y\alpha;$$
$$(\hat{\sigma}_x\hat{\sigma}_z-\hat{\sigma}_z\hat{\sigma}_x)\beta=-\hat{\sigma}_x\beta-\hat{\sigma}_z\alpha=-2\alpha=-2i\hat{\sigma}_y\beta$$

Şu iki aňlatmadan

$$\hat{\sigma}_x\hat{\sigma}_z-\hat{\sigma}_z\hat{\sigma}_x=-2i\hat{\sigma}_{y.}$$

Şuny-da subut etmelidi.

Mundan başga

$$\hat{\sigma}_y\hat{\sigma}_x\alpha=\hat{\sigma}_y\beta=-i\alpha=-i\hat{\sigma}_z\alpha,$$
$$\hat{\sigma}_y\hat{\sigma}_x\beta=\hat{\sigma}_y\alpha=i\beta=-i\hat{\sigma}_z\beta$$

Şu ikisinden

$$\hat{\sigma}_y\hat{\sigma}_x=-i\hat{\sigma}_z.$$

$\hat{\sigma}_x^2,\hat{\sigma}_y^2,\hat{\sigma}_z^2$ ululyklary hasaplap, alýarys

$$\hat{\sigma}_x^2=\hat{\sigma}_y^2=\hat{\sigma}_z^2=1$$

9.6. Aşakdaky operatorly gatnaşyklary subut etmeli:

a. $\hat{I}_x\hat{I}_y-\hat{I}_y\hat{I}_x=i\hbar\hat{I}_z.$

b. $\hat{I}_y\hat{I}_z-\hat{I}_z\hat{I}_y=i\hbar\hat{I}_x.$

ç. $\hat{I}_z\hat{I}_x-\hat{I}_x\hat{I}_z=i\hbar\hat{I}_y.$

d. $\hat{I}^2\hat{I}_x-\hat{I}_x\hat{I}^2=0.$

e. $\hat{I}^2\hat{I}_y-\hat{I}_y\hat{I}^2=0.$

ä. $\hat{I}^2\hat{I}_z-\hat{I}_z\hat{I}^2=0.$

f. $\hat I^2 \hat M^2 - \hat M^2 \hat I^2 = 0.$

g. $\hat I^2 \hat S^2 - \hat S^2 \hat I^2 = 0.$

Çözüwi:

a. $\hat I_x \hat I_y - \hat I_y \hat I_x = \left(\hat M_x + \hat S_x\right)\left(\hat M_y + \hat S_y\right) - \left(\hat M_y + \hat S_y\right)\left(\hat M_x + \hat S_x\right) = \hat M_x \hat M_y + \hat M_x \hat S_y + \hat S_x \hat M_y + \hat S_x \hat S_y -$

$- \hat M_y \hat M_x - \hat M_y \hat S_x - \hat S_y \hat M_x - \hat S_y \hat S_x = \left(\hat M_x \hat M_y - \hat M_y \hat M_x\right) + \left(\hat S_x \hat S_y - \hat S_y \hat S_x\right) = i\hbar \hat M_z + i\hbar \hat S_z =$

$= i\hbar\left(\hat M_z + \hat S_z\right) = i\hbar \hat I_z.$

d. $\hat I^2 \hat I_x - \hat I_x \hat I^2 = \left\{\hat M^2 + \hat S^2 + 2\left(\hat M_x \hat S_x + \hat M_y \hat S_y + \hat M_z \hat S_z\right)\right\}\left(\hat M_x + \hat S_x\right) - \left(\hat M_x + \hat S_x\right)\cdot$

$\cdot\left\{\hat M^2 + \hat S^2 + 2\left(\hat M_x \hat S_x + \hat M_y \hat S_y + \hat M_z \hat S_z\right)\right\} = 2\left(\hat M_x \hat S_x + \hat M_y \hat S_y + \hat M_z \hat S_z\right)\left(\hat M_x + \hat S_x\right) - 2\left(\hat M_x + \hat S_x\right)\cdot$

$\cdot\left(\hat M_x \hat S_x + \hat M_y \hat S_y + \hat M_z \hat S_z\right) = 2\left(\hat M_x \hat S_x \hat M_x + \hat M_x \hat S_x \hat S_x + \hat M_y \hat S_y \hat M_x + \hat M_y \hat S_y \hat S_x + \hat M_z \hat S_z \hat M_x + \hat M_z \hat S_z \hat S_x -\right.$

$\left.- \hat M_x \hat M_x \hat S_x - \hat M_x \hat M_y \hat S_y - \hat M_x \hat M_z \hat S_z - \hat S_x \hat M_x \hat S_x - \hat S_x \hat M_y \hat S_y - \hat S_x \hat M_z \hat S_z\right) = 2\left\{\left(\hat M_y \hat M_x - \hat M_x \hat M_y\right)\hat S_y +\right.$

$\left.+ \left(\hat S_y \hat S_x - \hat S_x \hat S_y\right)\hat M_y + \left(\hat M_z \hat M_x - \hat M_x \hat M_z\right)\hat S_z + \left(\hat S_z \hat S_x - \hat S_x \hat S_z\right)\hat M_z\right\} = 2\left(- i\hbar \hat M_z \hat S_y - i\hbar \hat S_z \hat M_y +\right.$

$\left.+ i\hbar \hat M_y \hat S_z + i\hbar \hat S_y \hat M_z\right) = 0,$

Sebäbi $\hat M_x$ operatory $\hat M^2$ we $\hat S^2$, $\hat S_x$ operatory hem $\hat M^2$ we $\hat S^2$ bilen we $\hat M$ we $\hat S$ operatorlary özara kommutirleşýärler.

X bap

Dirakyň matrisalarynyň algebrasy

X.1.Usuly görkezmeler

- α_n we ρ_n dörthatarly matrisalaryň toplumy ikihatarly matrisalar bilen aşakdaky gatnaşyklaryň üsti bilen baglanyşykly:

$$\alpha_n = \begin{pmatrix} \sigma_n' & 0' \\ 0' & \sigma_n' \end{pmatrix}; \qquad \rho_1 = \begin{pmatrix} 0' & I' \\ I' & 0' \end{pmatrix}; \rho_2 = \begin{pmatrix} 0' & -iI' \\ iI' & 0' \end{pmatrix}; \quad \rho_3 = \begin{pmatrix} I' & 0' \\ 0' & -I' \end{pmatrix},$$

- Dirakyň matrisalary:

$$\alpha_n = \rho_1 \sigma_n = \begin{pmatrix} 0' & \sigma_n' \\ \sigma_n' & 0' \end{pmatrix}$$

$$\alpha_0 = \rho_3 = \begin{pmatrix} I' & 0' \\ 0' & -I' \end{pmatrix}$$

bu ýerde $n = 1,2,3$ we σ_n' - Pauliniñ matrisalary:

$$\sigma_1' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2' = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Başga-da

$$O' = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \text{we} \qquad I' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- Dirakyň matrisalarynyň aýdyň görnüşleri:

$$\alpha_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \alpha_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \alpha_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

X.2. Meseleler

10.1. Aşakdaky gatnaşyklaryň dogrudyklaryny barlamaly.

a. $\alpha_n^2 = \rho_n^2 = I$;

b. $\alpha_1\alpha_2 = -\alpha_2\alpha_1 = i\alpha_3$ we başgalar.

ç. $\rho_1\rho_2 = -\rho_2\rho_1 = i\rho_3$ we başgalar.

Çözüwi:

$$\alpha_1^2 = \alpha_1 \cdot \alpha_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 1.$$

10.2. Aşakdaky gatnaşyklaryň dogrudyklaryny barlamaly.

a. $\alpha_1^2 = \rho_1^2\sigma_1^2 = 1,\qquad \alpha_0^2 = \rho_3^2 = I,$

b. $\alpha_2\alpha_3 + \alpha_3\alpha_2 = \rho_1^2(\sigma_2\sigma_3 + \sigma_3\sigma_2) = 0,$

ç. $\alpha_0\alpha_1 + \alpha_1\alpha_0 = \sigma_1(\rho_3\rho_1 + \rho_1\rho_3) = 0$ we başgalar.

Çözüwi:

b. $$\alpha_2\alpha_3 + \alpha_3\alpha_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} + \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} - \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} = 0.$$

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Mazmuny